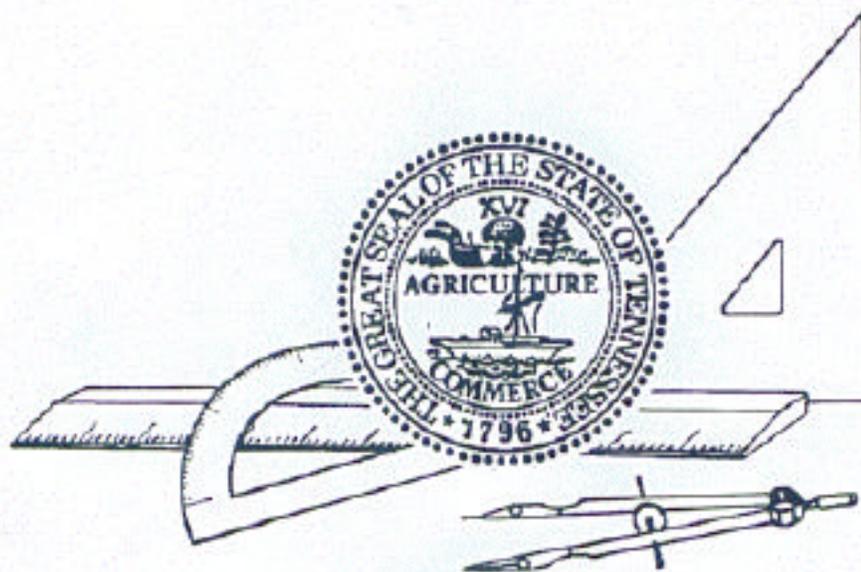


MATHEMATICS FRAMEWORK

GRADES KINDERGARTEN
THROUGH
GRADE EIGHT



STATE OF TENNESSEE
BOARD OF EDUCATION

**MATHEMATICS
CURRICULUM FRAMEWORK
GRADES K - 8**

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PREFACE

In accordance with a December 1995 recommendation by the Tennessee State Board of Education, the K-8 Mathematics Curriculum Framework has been reviewed and revised by a statewide committee of mathematics educators. The framework contains Process Standards, Content Standards, and Learning Expectations.

The foundation of this document is the four Process Standards: Problem Solving, Communication, Reasoning, and Connections. These four Process Standards are intended to be incorporated into each of the five Content Standards: Number Sense and Number Theory; Estimation, Measurement, and Computation; Patterns, Functions, and Algebraic Thinking; Statistics and Probability; and Spatial Sense and Geometric Concepts. The Learning Expectations identify the essential core of learning for grades K-2, 3-5, and 6-8 in each Content Standard.

This document contains the minimum expectations and shall be the basis for planning instructional programs at the local level. School systems have the flexibility to broaden their programs of study with regard to the depth and breadth of material to be covered at each grade level. The sequencing of objectives and the development of curriculum guides and lesson plans are the responsibility of system level instructional leaders.

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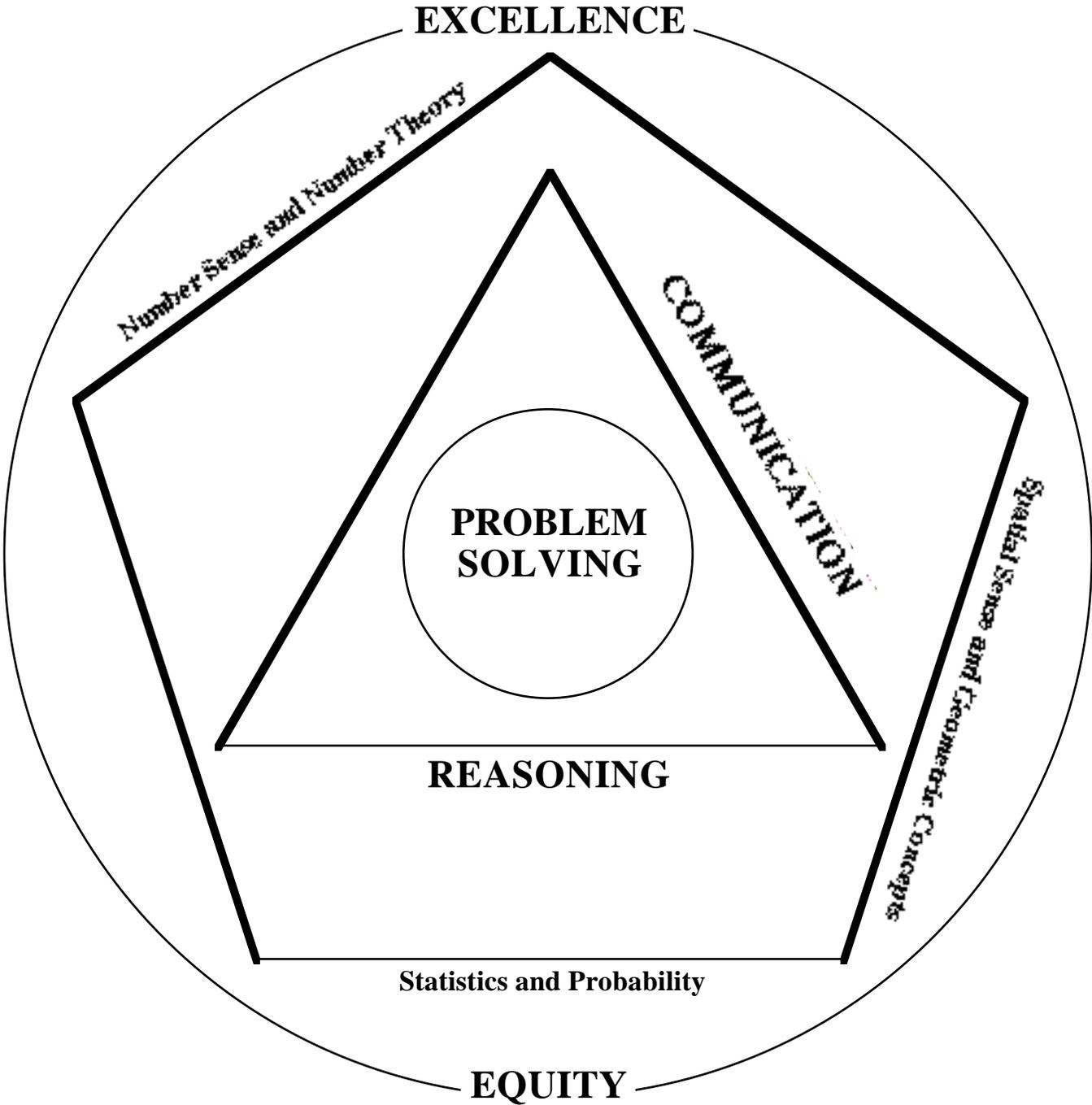
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MATHEMATICAL EMPOWERMENT OF STUDENTS



Empowering all students mathematically is the focus of this document.

PHILOSOPHY

The rationale for updating the Tennessee Mathematics Curriculum Framework for kindergarten through eighth grade is to improve the public education provided to children. Empowering all students mathematically is the focus of this document. The vision of mathematics education described in recent systemic reform documents, most notably the National Council of Teachers of Mathematics *Curriculum and Evaluation Standards for School Mathematics*, is grounded in extensive robust research on how children learn mathematics. Thus, those documents provide the basis of the Framework. At the heart of all recommendations in this document are the *New Goals for Students*.

1. **Learning to value mathematics.** Students should have numerous and varied experiences related to the cultural, historical, and scientific evolution of mathematics so that they can appreciate the role of mathematics in the development of our contemporary society and explore relationships among mathematics and the disciplines it serves: the physical and life sciences, the social sciences, and the humanities. . . . It is the intent of this goal to focus attention on the need for student awareness of the interaction between mathematics and the historical situations from which it has developed and the impact that interaction has on our culture and our lives.
2. **Becoming confident in one's own ability.** As a result of studying mathematics, students need to view themselves as capable of using their growing mathematical power to make sense of new problem situations in the world around them. To some extent everyone is a mathematician and does mathematics consciously. To buy at the market, to measure a strip of wallpaper, or to decorate a ceramic pot with a regular pattern is doing mathematics. School mathematics must endow all students with the realization that doing mathematics is a common human activity. Having numerous and varied experiences allows students to trust their own mathematical thinking.
3. **Becoming a mathematical problem solver.** The development of each student's ability to solve problems is essential if he or she is to be a productive citizen. "Problem solving must be the focus of school mathematics" (NCTM, 1980). To develop such abilities, students need to work on problems that may take hours, days, and even weeks to solve. Although some may be relatively simple

exercises to be accomplished independently, others should involve small groups or an entire class working cooperatively. Some problems should also be open-ended with no single right answer, while others need to be formulated.

4. **Learning to communicate mathematically.** The development of a student's power to use mathematics involves learning the signs, symbols, and terms of mathematics. This is best accomplished in problem situations in which students have an opportunity to read, write, and discuss ideas in which the use of the language of mathematics becomes natural. As students communicate their ideas, they learn to clarify, refine, and consolidate their answers.
5. **Learning to reason mathematically.** Making conjectures, gathering evidence, and building an argument to support such notions are fundamental to doing mathematics. In fact, a demonstration of good reasoning should be rewarded even more than students' ability to find correct answers.

New Goals for Students (Reprinted with permission from *Curriculum and Evaluation Standards for School Mathematics*, 1989 by the National Council of Teachers of Mathematics. All rights reserved.)

In summary, the intent of these goals is that students will become mathematically literate. This term denotes an individual's ability to explore, to conjecture, and to reason logically, as well as to use a variety of mathematical methods effectively to solve problems. By becoming literate, their mathematical power should develop.

Traditionally, prescribed algorithms for pencil-and-paper computation have occupied a substantial amount of students' time in elementary school mathematics. To empower students mathematically, the emphasis must shift away from rote symbol manipulation and memorization without understanding and must move toward emphasizing mathematics as problem solving, as reasoning, and as communication. Furthermore, students should explore and build upon the connections among mathematical concepts, the connections between mathematics and other disciplines, and the mathematics in everyday life. To that end, this Framework describes four Process Standards: (1) problem solving, (2) reasoning, (3) communication, and (4) connections. These four Process Standards permeate the Content Standards. This document acknowledges that teaching any particular content well involves creating an environment in

which students are engaged in significant problem solving, in sense-making, in mathematical discourse, and in recognizing inter-relationships.

The issue of equity, particularly in mathematics education, is crucial. Mathematics is a filter for employment. That is, lack of mathematical background eliminates job opportunities. Therefore, it is vital that all students, especially females and minorities who have traditionally been underrepresented in mathematics-intensive fields, be strongly supported in mathematics education.

Emphasis on pencil-and-paper algorithms is inadequate for today's world. Because calculators and computers are widely accessible, the nature of the problems of mathematics and the methods used to investigate them have changed tremendously. Indeed, technology is pervasive in industry and business. Students now need to know how to use a variety of tools, including technology and mental calculations, as well as pencil and paper.

Tennessee joins other states in basing the State Curriculum Framework on systemic reform. In fact the prior work of committees in other states, such as Delaware, Georgia, and California, has made our task less daunting. It is the belief of this committee that implementation of the recommendations stated in this document will enable Tennessee to improve the mathematics education of its students. This committee understands that the changes delineated here are not merely adjustments of the former curriculum. Instead, this document represents systemic change in mathematics education that will require extensive staff development and necessitate new forms of assessment that reflect the emphasis on higher order thinking.

HISTORY

The Basic Skills First curriculum initiatives that began in the late 1970's involved elementary and middle school teachers and mathematics curriculum specialists in identifying by grade level, K-8, the skills and concepts that should be taught. This effort resulted in the first statewide mathematics curriculum document which, with later revisions, became the Blue Book. Key skills at the 6th-8th grade level were identified as minimum proficiency skills and policy was adopted which required students to pass a test based on these skills in order to receive a high school diploma. The list of proficiency skills has been revised several times since the late 1970's. In 1993, another revision occurred which resulted in the TCAP Competency Test. The Competency Test is based on 8th grade skills as identified in the state framework. Passing it remains a graduation requirement in order for students to receive a high school diploma.

In 1984 the Comprehensive Education Reform Act (CERA) was passed by the Tennessee Legislature with the stipulation that within five years instructional programs should be improved in a measurable way. Mathematics, as described in Academic Preparation for College: What Students Need to Know and Be Able to Do (the Green Book), was included as a subject to show measurable improvement. In 1986 the Tennessee State-Wide School-College Collaborative for Education Excellence (the Collaborative) was organized and included several Task Forces, including one for Mathematics. The Mathematics Task Force looked at the existing State Mathematics curriculum documents and produced the two documents, Mathematics Framework (K-8) and Mathematics Curriculum Guide (9-12). These documents were correlated to the Green Book as required by CERA.

In 1989 the NCTM Curriculum and Evaluation Standards for School Mathematics was published. A cadre of mathematics educators from across the state, including K-16 representation, was assembled to receive training for leaders offered by NCTM and to then conduct awareness workshops across the state regarding the Standards. In addition, this cadre reviewed the K-8 and 9-12 frameworks in light of the Standards and made some revisions of those documents in 1991. Also in 1991 Math for Technology I was added to the state curriculum for 9-12 and the Professional Standards for Teaching Mathematics was published by NCTM.

In 1992 the Tennessee Legislature passed the Education Improvement Act based on the 21st Century Challenge Plan of 1990. As a result, several initiatives began. The Tennessee Comprehensive Curriculum Guide, Grades K-8 was assembled. It included skills and concepts for Language Arts, Mathematics, Science, and Social Studies for these grades in one document that became known as the Blue Book. Also, the State Board of Education adopted a Mathematics Policy endorsing the NCTM Curriculum and Evaluation Standards for School Mathematics and several mathematics professional groups and individuals across the state began efforts to review the existing frameworks and guides in light of the Standards. One of these groups was formed by the Systemic Initiative Steering Committee and received funding from the Eisenhower Math/Science Consortium at Appalachia Educational Laboratory to prepare a companion document to the frameworks that would connect these two frameworks with the NCTM standards documents. The document, Mathematics for All Tennessee Students, was written in 1994 with the goal of making the frameworks more useful to teachers and more compatible with the Standards.

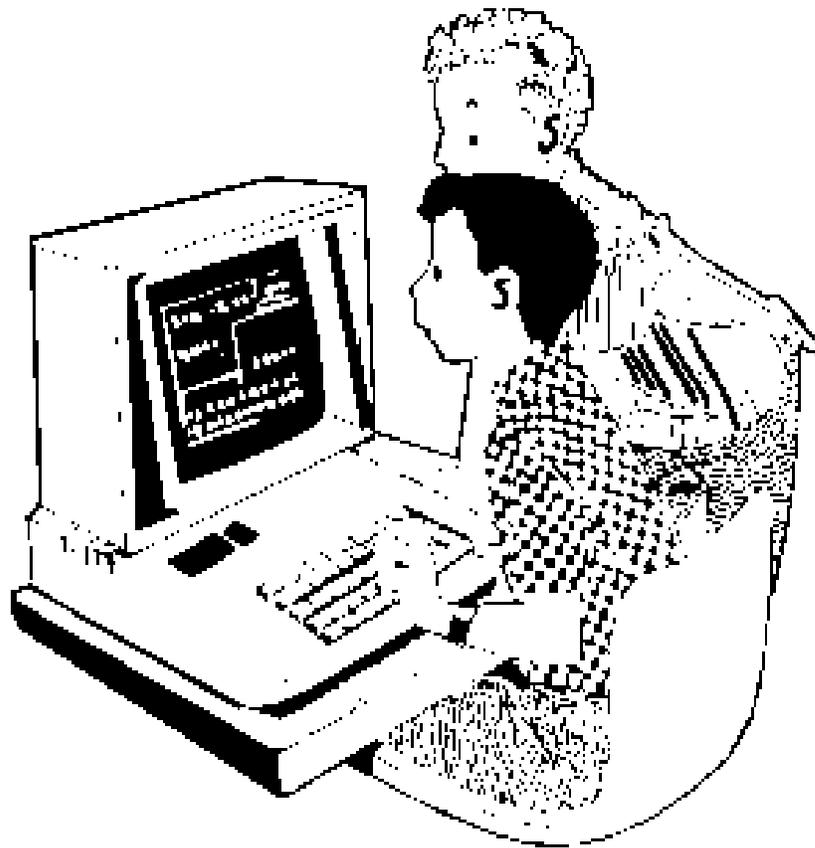
In 1993 Math for Technology II was approved as a high school course and was certified by governing bodies as meeting the Algebra I admission requirement for state universities.

In 1994 the high school policy requiring the graduation requirement of 3 years of mathematics including Algebra I or the equivalent, Math for Technology II, was implemented.

In 1995 preparation for instituting End of Course Assessments in high school mathematics courses began. Task forces of teachers from across the state were convened to write a bank of items in all the existing mathematics courses included in the existing curriculum framework. The items were submitted to CTB/McGraw Hill to be used to construct subject area tests for Pre-Algebra, Math for Technology I, Algebra I, Geometry, and Algebra II. Preparation for tests for other mathematics courses was delayed.

In the spring of 1996 a team of mathematics teachers and curriculum specialists representing K-16 and the entire state was convened to rewrite the K-8 state frameworks based on the Standards and current practice. The revision was then reviewed by mathematics educators statewide. This effort resulted in the production of this document which identifies the Process Standards, Content Standards, and Learning Expectations that should guide state school systems in making the changes needed to improve the learning of mathematics by students across the state.

PROCESS STANDARDS



MATHEMATICS AS PROBLEM SOLVING

The study of mathematics must emphasize **Problem Solving** opportunities which require various approaches to investigate, understand, and apply mathematical concepts.

The development of each learner's ability to solve problems is essential if he or she is to be a productive citizen. We strongly endorse the first recommendation of *An Agenda for Action* (NCTM, 1980): "Problem solving must be the focus of school mathematics." To develop such abilities, students need to work on problems that may take hours, days, and even weeks to solve. Some may be relatively simple exercises to be accomplished independently; some should involve small groups or an entire class working cooperatively; and some problems should also be open-ended with no single right answer.

"Mathematics as Problem Solving" emphasizes the learners' use of strategies to:

- Investigate and understand mathematical content.
- Recognize and formulate problems.
- Solve a wide variety of problems.
- Generalize solutions and strategies, applying them to new problems.
- Acquire confidence in their ability to use mathematics meaningfully (NCTM, 1996).

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MATHEMATICS AS COMMUNICATION

The study of mathematics must emphasize **Communication** by requiring opportunities to explain, conjecture, and defend one's ideas orally, in writing, and through the use of technology.

The development of a learner's power to use mathematics involves learning the signs, symbols, and terms of mathematics. This is best accomplished in problem situations in which students have an opportunity to read, write, and discuss ideas in which the use of the language of mathematics becomes natural. As students communicate their ideas, they learn to clarify, refine, and consolidate their answers.

“Mathematics as Communication” focuses on the learners' development of using language and symbols to:

- Clarify thinking about mathematical ideas and situations.
- Express mathematical ideas orally, in writing, and with physical material, pictures, and diagrams.
- Understand and value the role of mathematical notation.
- Realize that representing, discussing, listening, writing, and reading mathematics are vital aspects of mathematics study and use.

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MATHEMATICS AS REASONING

The study of mathematics must emphasize **Reasoning** which requires critical thinking, logical arguments, and justification of solutions, of thought processes, and of conjectures.

Making conjectures, gathering evidence, and building an argument to support such notions are fundamental to doing mathematics. In fact, a demonstration of good reasoning should be rewarded even more than the learner's ability to find correct answers.

“Mathematics as Reasoning” concentrates on leading the learners to:

- Draw logical conclusions about mathematics.
- Use mathematical models, facts, properties, and relationships to explain their thinking.
- Justify solution-finding processes and answers.
- Use patterns and relationships to analyze mathematical situations.
- Make and evaluate mathematical conjectures.
- Believe that mathematics makes sense.

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MATHEMATICAL CONNECTIONS

The study of mathematics must emphasize making **Connections** among the various topics within mathematics, between mathematics and other disciplines, and between mathematics and “real world” situations.

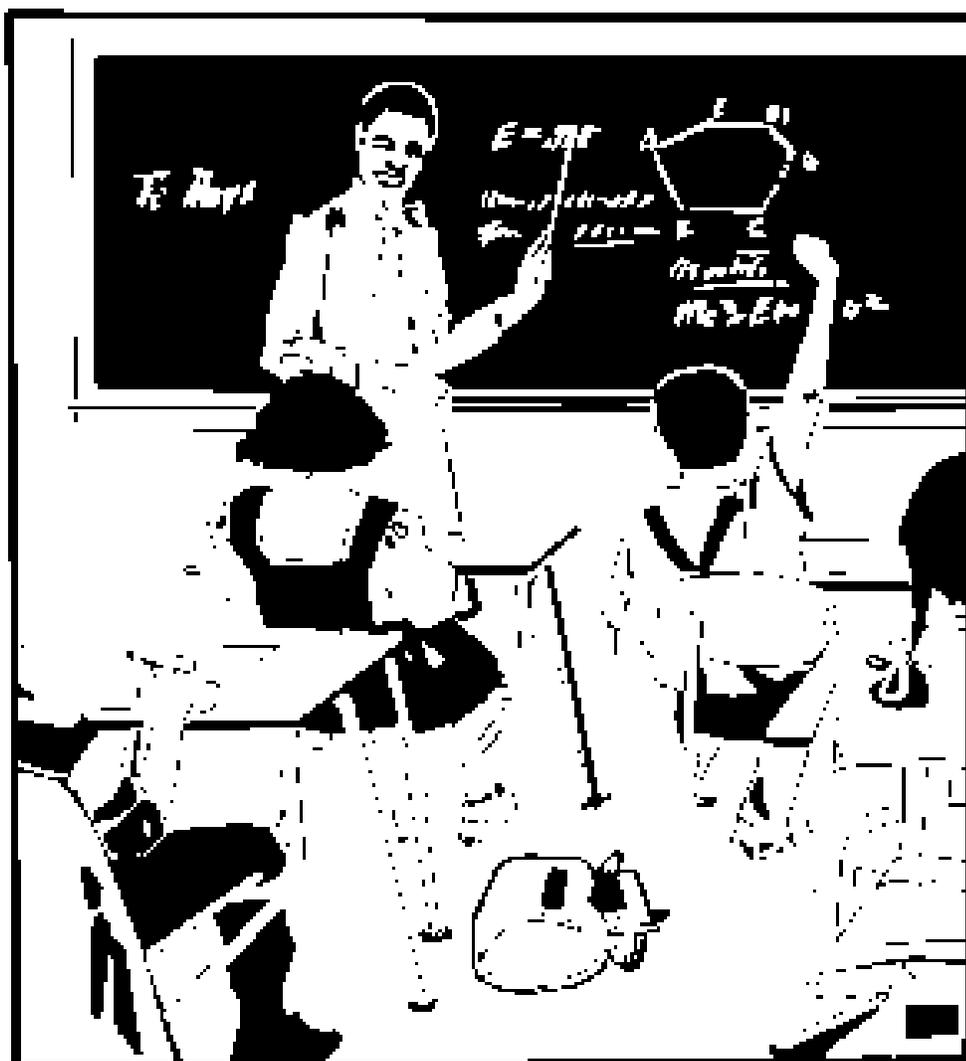
The mathematics curriculum is often viewed as consisting of several discrete strands; so topics tend to be taught in isolation. Unless the learners connect ideas both among and between areas of mathematics, they learn isolated skills rather than develop the ability to recognize general principles and procedures relevant to several areas. Connecting conceptual understanding to procedures will enable learners to apply, recreate, and invent new procedures when needed. Failure to connect conceptual understanding to procedures results in a view of mathematics as an arbitrary set of rules. Learners should have many opportunities to observe and work with the interaction of mathematics with other school subjects and with everyday society. Problems become real-world and meaningful when they relate to the learners’ experiences. Mathematics must be integrated into contexts that give its symbols and processes practical meaning. The school environment is rich with opportunities to use mathematics in other subject areas as well as other subject area content in mathematics.

“Mathematical Connections” concentrate on enabling the learners to:

- See mathematics as an integrated whole, linking conceptual and procedural knowledge within the discipline and relating multiple representations of concepts or procedures to one another.
- Apply mathematical thinking and modeling to solve problems that arise in other disciplines and curriculum areas, such as art, business, music, psychology, industrial arts, computer technology, social studies, and sciences, such as biology, chemistry, and physics.
- Use, recognize, and value the varied roles of mathematics in their lives, culture, and society.

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CONTENT STANDARDS



NUMBER SENSE AND NUMBER THEORY

In order to develop **Number Sense** and an understanding of **Number Theory**, the mathematics curriculum must include problems which require students to recognize, represent, and model real numbers and operations verbally, physically, symbolically, and graphically.

K - 2

Learning Expectations

Through the investigation of meaningful problems, individually or in cooperative groups while using appropriate technology, all students in grades K-2 will be able to:

- count sets of objects individually or by grouping;
- relate the use of number in the learners' environment to the study of number;
- represent whole numbers and fractions concretely, pictorially, and symbolically;
- demonstrate an understanding of the relative magnitude of numbers;
- demonstrate a sense of reasonableness of results involving number and measurement;
- use known number relationships to discover new relationships;
- apply understanding of relationships between operations;
- demonstrate understanding of whole numbers by relating counting, grouping, and place value concepts.

3 - 5

Learning Expectations

Through the investigation of meaningful problems, individually or in cooperative groups while using appropriate technology, all students in grades 3-5, building upon the K-2 expectations, will be able to:

- demonstrate an understanding of whole numbers, fractions, and decimals through the use of technology and physical, pictorial, and symbolic representations;

Effective Teaching of Number Sense and Number Theory involves creating an environment in which the learners are engaged in significant problem solving, in reasoning, in communication, and in making connections.

- demonstrate an understanding of order relationships for fractions, decimals, and whole numbers using physical, verbal, and symbolic representations;
- articulate and model the relationship between fractions and decimal notation for a rational number;
- select a notation and justify its appropriateness for a particular situation;
- recognize and apply the properties of whole numbers and their operations;
- apply and explain number theory concepts (e.g., primes, factors, multiples) in problem situations;
- demonstrate an understanding of decimals by extending whole number place value concepts;
- articulate the relationships among the four arithmetic operations;
- demonstrate understanding of equivalent forms of rational numbers;
- apply the study of whole numbers and fractions to situations in the learners' environment;
- use geometric models to illustrate properties of whole numbers and fractions and their operations.

6 - 8

Learning Expectations

Through the investigation of meaningful problems, individually or in cooperative groups while using appropriate technology, all students in grades 6-8, building upon K-5 expectations, will be able to:

- represent and use numbers in a variety of equivalent forms, including integers, fractions, decimals, percent, and exponential and scientific notation;
- articulate and model the relationship among decimals, fraction, ratios and percents;
- model integers and their operations;
- model various interpretations, including ratio, of rational numbers

Effective Teaching of Number Sense and Number Theory involves creating an environment in which the learners are engaged in significant problem solving, in reasoning, in communication, and in making connections.

ESTIMATION, MEASUREMENT, AND COMPUTATION

In order to develop understanding of **Estimation, Measurement, and Computation**, the mathematics curriculum should include problems which require students to apply appropriate tools and units of measurement, to develop effective estimation and computation strategies for producing reasonable results, and to calculate using appropriate tools such as mental mathematics, technology, manipulatives, and pencil-and-paper.

K - 2

Learning Expectations

Through the investigation of meaningful problems, individually or in cooperative groups while using appropriate technology, all students in grades K-2 will be able to:

- apply estimation strategies in working with quantities, measurements, computations, and problem solving;
- recognize when estimating is appropriate;
- demonstrate the reasonableness of solutions and communicate a supporting statement;
- demonstrate, using standard and non-standard units, the attributes of length, capacity, weight, area, time, money, temperature, and angle;
- demonstrate the meaning for operations by modeling and discussing a wide variety of problems;
- relate the mathematical language and symbolism of operations to problem situations and informal language;
- demonstrate thinking strategies for basic number facts; select and use appropriate computational operations and methods.

Effective Teaching of Estimation, Measurement, and Computation involves creating an environment in which the learners are engaged in significant problem solving, in reasoning, in communication, and in making connections.

3 - 5

Learning Expectations

Through the investigation of meaningful problems, individually or in cooperative groups while using appropriate technology, all students in grades 3-5, building upon the K-2 expectations, will be able to:

- use physical models to demonstrate the relationships among numbers and operations including fractions and decimals;
- use a variety of measuring devices to collect, organize, and communicate data;
- apply estimation strategies;
- demonstrate with manipulative materials the relationships among perimeter, area, and volume;
- estimate, calculate, and evaluate solutions and communicate a supporting statement.

6 - 8

Learning Expectations

Through the investigation of meaningful problems, individually or in cooperative groups while using appropriate technology, all students in grades 6-8, building upon K-5 expectations, will be able to:

- compute and estimate with rational numbers including whole numbers, fractions, decimals, and integers;
- explain the concepts underlying computational procedures involving rational numbers, including whole numbers, fractions, decimals, and integers;
- demonstrate understanding of the concepts of perimeter, area, volume, angle measure, capacity, weight and mass, and of any related formulas;
- apply formulas and explain strategies for solving problems related to measurement;
- demonstrate an understanding of rates and other derived and indirect measurements;
- select and apply an appropriate method for computing from among mental arithmetic, paper and pencil, and appropriate technology and evaluate the reasonableness of results.

Effective Teaching of Estimation, Measurement, and Computation involves creating an environment in which the learners are engaged in significant problem solving, in reasoning, in communication, and in making connections.

PATTERNS, FUNCTIONS, AND ALGEBRAIC THINKING

In order to develop **Algebraic Thinking** through an understanding of **Patterns and Functions**, the mathematics curriculum must include problems which require students to describe, extend, analyze, and create a wide variety of patterns and functions using appropriate materials and representations such as tables, graphs, and mathematical notation.

K - 2

Learning Expectations

Through the investigation of meaningful problems, individually or in cooperative groups, while using appropriate technology, all students in grades K-2 will be able to:

- sort and classify objects by common attributes;
- sort numbers into different classes, such as even, odd, greater than 10, or multiple of 6;
- demonstrate understanding of the meaning of number sentences (equalities written either horizontally or vertically as well as inequalities);
- solve open sentences using informal methods;
- describe, extend, and create a wide variety of visual, oral, and physical patterns, including those found in real life and in other disciplines, such as literature and music;
- describe mathematical relationships, including those involving number and geometric objects.

3 - 5

Learning Expectations

Through the investigation of meaningful problems, individually or in cooperative groups while using appropriate technology, all students in grades 3-5, building upon the K-2 expectations, will be able to:

- describe and extend patterns and functions represented in tables and graphs;

Effective Teaching of Patterns, Functions, and Algebraic Thinking involves creating an environment in which the learners are engaged in significant problem solving, in reasoning, in communication, and in making connections.

- solve open sentences using informal methods and knowledge of operations;
- use tables and graphs to identify properties and relationships;
- use functions and patterns to represent and solve problems;
- describe and represent functions found in a variety of situations with tables, graphs, verbal rules, and equations using variables.

6 - 8

Learning Expectations

Through the investigation of meaningful problems, individually or in cooperative groups while using appropriate technology, all students in grades 6-8, building upon K-5 expectations, will be able to:

- describe, extend, analyze, and create a wide variety of patterns in numbers, shape, and data using a variety of appropriate materials, including manipulatives and technology;
- apply understanding of functional relationships to explain how a change in one quantity results in a change in another;
- use mathematical language and symbols to describe patterns and represent functional relationships;
- identify patterns and functions from statistical data;
- use patterns and functions to represent and solve problems;
- demonstrate an understanding of the concepts of variable, expression, and equation;
- represent situations with tables, graphs, verbal rules, and equations and describe the interrelationships of the representations;
- evaluate algebraic expressions and formulas for given values of the variable using manipulatives, technology and pencil/paper;
- solve linear equations using concrete, informal and formal methods;
- describe and use informal methods to solve linear inequalities and non-linear equations;
- model and solve problems using algebraic methods.

Effective Teaching of Patterns, Functions, and Algebraic Thinking involves creating an environment in which the learners are engaged in significant problem solving, in reasoning, in communication, and in making connections.

STATISTICS AND PROBABILITY

In order to develop an understanding of **Statistics and Probability**, the mathematics curriculum must include problems which require students to collect, organize, represent, and interpret data; make inferences and predictions; present and evaluate inferences and predictions; present and evaluate arguments based on data analysis; and model situations to determine theoretical and experimental probabilities.

K - 2

Learning Expectations

Through the investigation of meaningful problems, individually or in cooperative groups while using appropriate technology, students in K-2 should be able to:

- collect data by observing, measuring, surveying and counting;
- demonstrate a variety of techniques for representing and organizing data such as arrays of physical objects, tallies, pictographs and bar graphs;
- interpret data by identifying patterns and relationships, considering cause and effect, and drawing conclusions;
- articulate specific questions based on collected data;
- determine and justify the relative likelihood of simple chance events.

3 - 5

Learning Expectations

Through the investigation of meaningful problems, individually or in cooperative groups while using appropriate technology and employing equity for all, all students in grades 3-5, building upon the K-2 expectations, will be able to:

- construct and use appropriate representations of data through data collections and organizations a variety of displays including line plots, tables, histograms, scale pictographs, and charts;

Effective Teaching of Statistics and Probability involves creating an environment in which the learners are engaged in significant problem solving, in reasoning, in communication, and in making connections.
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- interpret data and make convincing arguments that are based on data analysis, including mean, median and mode;
- represent all possible outcomes for an experiment by using a tree diagram;
- determine the probability of a single event based on an experiment with equally likely outcomes;
- formulate and solve problems that involve collecting and analyzing data.

6 - 8

Learning Expectations

Through the investigation of meaningful problems, individually or in cooperative groups while using appropriate technology, and while building upon the K-5 expectations, students in 6-8 should be able to:

- collect, organize, and describe data in order to make conjectures;
- formulate and test hypotheses, draw conclusions, and make convincing arguments that are based on data analysis;
- construct, interpret, and use various displays of data to answer questions and solve problems;
- calculate and use range and measures of central tendency to describe data;
- determine experimental probability by devising and carrying out experiments or simulations;
- make conjectures based on experimental or theoretical probabilities;
- analyze a sample to make inferences about a population;
- construct a sample space to determine the theoretical probability.

Effective Teaching of Statistics and Probability involves creating an environment in which the learners are engaged in significant problem solving, in reasoning, in communication, and in making connections.

SPATIAL SENSE AND GEOMETRIC CONCEPTS

In order to develop **Spatial Sense** and an understanding of **Geometric Concepts**, the mathematics curriculum must include problems which require students to explore geometric properties and relationships and to investigate, model, and analyze one-, two-, and three-dimensional figures.

K - 2

Learning Expectations

Through the investigation of meaningful problems, individually or in cooperative groups, while using appropriate technology, all students in grades K-2 will be able to:

- identify and classify solid and plane figures by attributes such as shape, size, symmetry, edges, corners, and faces;
- articulate the relationships between geometric concepts and objects in the real world;
- model geometric shapes using a variety of materials;
- create new figures by combining shapes and identify components of complex shapes;
- communicate geometric and directional descriptions effectively;
- model flips, slides, and turns.

3 - 5

Learning Expectations

Through the investigation of meaningful problems, individually or in cooperative groups while using appropriate technology, all students in grades 3-5, building upon K-2 expectations, will be able to:

- identify, compare, and accurately represent plane figures using a variety of materials;
- describe and compare properties of polygons such as number of sides, number of vertices, classification of angles and parallel or perpendicular sides;
- apply and explain geometric properties and relationships such as congruency, similarity, and symmetry;

Effective Teaching of Spatial Sense and Geometric Concepts involves creating an environment in which the learners are engaged in significant problem solving, in reasoning, in communication, and in making connections.
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- identify and construct models of three dimensional figures both with and without nets;
- demonstrate transformations using models and pictorial representations;
- apply and compare concepts of area, perimeter, circumference, and volume.

6 - 8

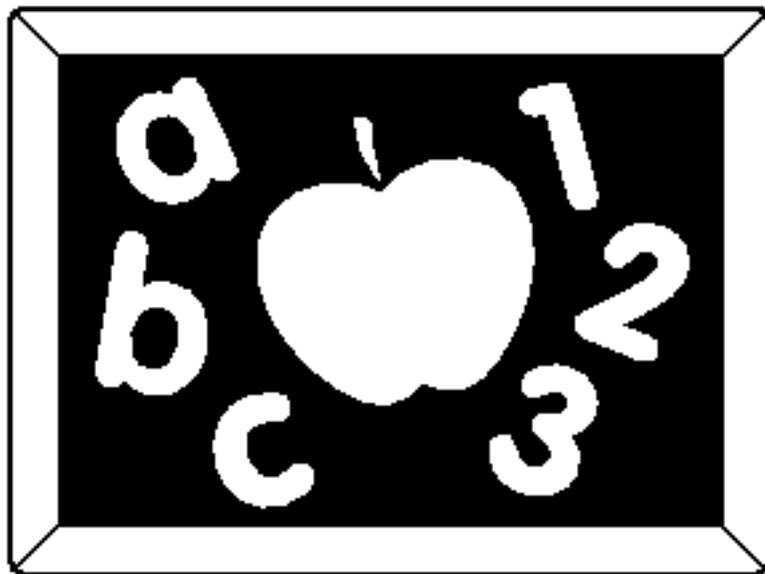
Learning Expectations

Through the investigation of meaningful problems, individually or in cooperative groups while using appropriate technology, all students in grades 6-8, building upon the K-5 expectations, will be able to:

- demonstrate geometric relationships through the use of manipulatives, graphing calculators, computers, and construction tools such as compass, straight edge and protractor;
- apply and explain effects of basic transformations on geometric figures;
- apply geometric properties and relationships to make conjectures;
- articulate geometric properties and relationships such as congruence, similarity, parallelism, perpendicularity, and symmetry;
- apply geometric concepts to problems in other fields and disciplines;
- represent and solve problems using geometric models.

<p>Effective Teaching of Spatial Sense and Geometric Concepts involves creating an environment in which the learners are engaged in significant problem solving, in reasoning, in communication, and in making connections.</p>

K - 2



K-2

NUMBER SENSE AND NUMBER THEORY

In order to develop **Number Sense** and an understanding of **Number Theory**, the mathematics curriculum must include problems which require students to recognize, represent, and model real numbers and operations verbally, physically, symbolically, and graphically.

Learning Expectations

Through the investigation of meaningful problems, individually or in cooperative groups while using appropriate technology, all students in grades K-2 will be able to:

- count sets of objects individually or by grouping;
- relate the use of number in the learners' environment to the study of number;
- represent whole numbers and fractions concretely, pictorially, and symbolically;
- demonstrate an understanding of the relative magnitude of numbers;
- demonstrate a sense of reasonableness of results involving number and measurement;
- use known number relationships to discover new relationships;
- apply understanding of relationships between operations;
- demonstrate understanding of whole numbers by relating counting, grouping, and place value concepts.

ESTIMATION, MEASUREMENT, AND COMPUTATION

In order to develop understanding of **Estimation, Measurement, and Computation**, the mathematics curriculum should include problems which require students to apply appropriate tools and units of measurement, to develop effective estimation and computation strategies for producing reasonable results, and to calculate using appropriate tools such as mental mathematics, technology, manipulatives, and pencil-and-paper.

Learning Expectations

Through the investigation of meaningful problems, individually or in cooperative groups while using appropriate technology, all students in grades K-2 will be able to:

- apply estimation strategies in working with quantities, measurements, computations, and problem solving;
- recognize when estimating is appropriate;
- demonstrate the reasonableness of solutions and communicate a supporting statement;
- demonstrate, using standard and non-standard units, the attributes of length, capacity, weight, area, time, money, temperature, and angle;
- demonstrate the meaning for operations by modeling and discussing a wide variety of problems;
- relate the mathematical language and symbolism of operations to problem situations and informal language;
- demonstrate thinking strategies for basic number facts; select and use appropriate computational operations and methods.

PATTERNS, FUNCTIONS, AND ALGEBRAIC THINKING

In order to develop **Algebraic Thinking** through an understanding of **Patterns and Functions**, the mathematics curriculum must include problems which require students to describe, extend, analyze, and create a wide variety of patterns and functions using appropriate materials and representations such as tables, graphs, and mathematical notation.

Learning Expectations

Through the investigation of meaningful problems, individually or in cooperative groups, while using appropriate technology, all students in grades K-2 will be able to:

- sort and classify objects by common attributes;
- sort numbers into different classes, such as even, odd, greater than 10, or multiple of 6;
- demonstrate understanding of the meaning of number sentences (equalities written either horizontally or vertically as well as inequalities);
- solve open sentences using informal methods;

- describe, extend, and create a wide variety of visual, oral, and physical patterns, including those found in real life and in other disciplines, such as literature and music;
- describe mathematical relationships, including those involving number and geometric objects.

STATISTICS AND PROBABILITY

In order to develop an understanding of **Statistics and Probability**, the mathematics curriculum must include problems which require students to collect, organize, represent, and interpret data; make inferences and predictions; present and evaluate inferences and predictions; present and evaluate arguments based on data analysis; and model situations to determine theoretical and experimental probabilities.

Learning Expectations

Through the investigation of meaningful problems, individually or in cooperative groups while using appropriate technology, students in K-2 should be able to:

- collect data by observing, measuring, surveying and counting;
- demonstrate a variety of techniques for representing and organizing data such as arrays of physical objects, tallies, pictographs and bar graphs;
- interpret data by identifying patterns and relationships, considering cause and effect, and drawing conclusions;
- articulate specific questions based on collected data;
- determine and justify the relative likelihood of simple chance events.

SPATIAL SENSE AND GEOMETRIC CONCEPTS

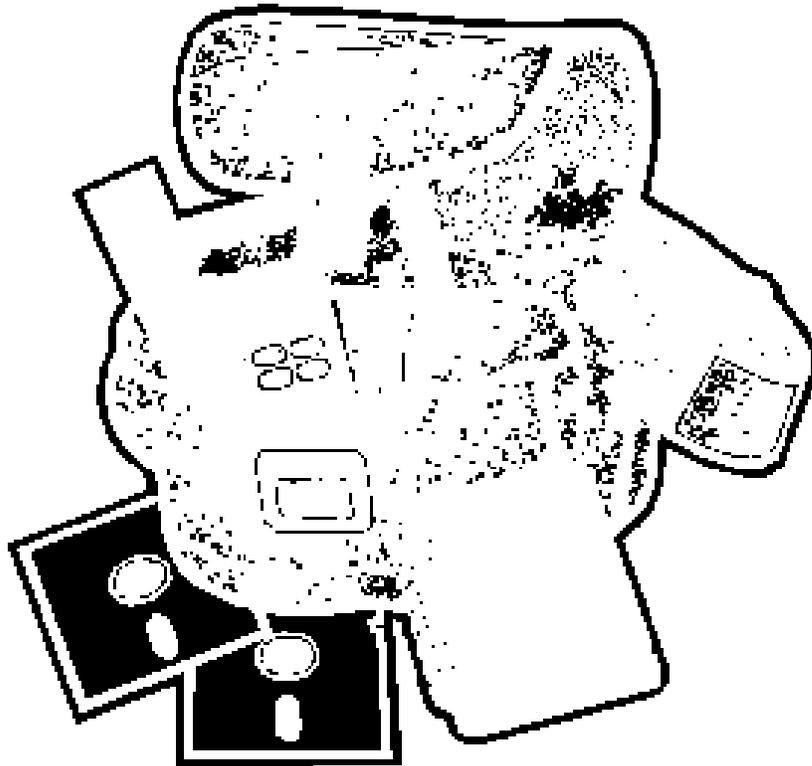
In order to develop **Spatial Sense** and an understanding of **Geometric Concepts**, the mathematics curriculum must include problems which require students to explore geometric properties and relationships and to investigate, model, and analyze one-, two-, and three-dimensional figures.

Learning Expectations

Through the investigation of meaningful problems, individually or in cooperative groups, while using appropriate technology, all students in grades K-2 will be able to:

- identify and classify solid and plane figures by attributes such as shape, size, symmetry, edges, corners, and faces;
- articulate the relationships between geometric concepts and objects in the real world;
- model geometric shapes using a variety of materials;
- communicate geometric and directional descriptions effectively;
- model flips, slides, and turns.

3 - 5



3 - 5

NUMBER SENSE AND NUMBER THEORY

In order to develop **Number Sense** and an understanding of **Number Theory**, the mathematics curriculum must include problems which require students to recognize, represent, and model real numbers and operations verbally, physically, symbolically, and graphically.

Learning Expectations

Through the investigation of meaningful problems, individually or in cooperative groups while using appropriate technology, all students in grades 3-5, building upon the K-2 expectations, will be able to:

- demonstrate an understanding of whole numbers, fractions, and decimals through the use of technology and physical, pictorial, and symbolic representations;
- demonstrate an understanding of order relationships for fractions, decimals, and whole numbers using physical, verbal, and symbolic representations;
- articulate and model the relationship between fractions and decimal notation for a rational number;
- select a notation and justify its appropriateness for a particular situation;
- recognize and apply the properties of whole numbers and their operations;
- apply and explain number theory concepts (e.g., primes, factors, multiples) in problem situations;
- demonstrate an understanding of decimals by extending whole number place value concepts;
- articulate the relationships among the four arithmetic operations;
- demonstrate understanding of equivalent forms of rational numbers;
- apply the study of whole numbers and fractions to situations in the learners' environment;
- use geometric models to illustrate properties of whole numbers and fractions and their operations.

ESTIMATION, MEASUREMENT, AND COMPUTATION

In order to develop understanding of **Estimation, Measurement, and Computation**, the mathematics curriculum should include problems which require students to apply appropriate tools and units of measurement, to develop effective estimation and computation strategies for producing reasonable results, and to calculate using appropriate tools such as mental mathematics, technology, manipulatives, and pencil-and-paper.

Learning Expectations

Through the investigation of meaningful problems, individually or in cooperative groups while using appropriate technology, all students in grades 3-5, building upon the K-2 expectations, will be able to:

- use physical models to demonstrate the relationships among numbers and operations including fractions and decimals;
- use a variety of measuring devices to collect, organize, and communicate data;
- apply estimation strategies;
- demonstrate with manipulative materials the relationships among perimeter, area, and volume;
- estimate, calculate, and evaluate solutions and communicate a supporting statement.

PATTERNS, FUNCTIONS, AND ALGEBRAIC THINKING

In order to develop **Algebraic Thinking** through an understanding of **Patterns and Functions**, the mathematics curriculum must include problems which require students to describe, extend, analyze, and create a wide variety of patterns and functions using appropriate materials and representations such as tables, graphs, and mathematical notation.

Learning Expectations

Through the investigation of meaningful problems, individually or in cooperative groups while using appropriate technology, all students in grades 3-5, building upon the K-2 expectations, will be able to:

- describe and extend patterns and functions represented in tables and graphs;
- solve open sentences using informal methods and knowledge of operations;
- use tables and graphs to identify properties and relationships;
- use functions and patterns to represent and solve problems;
- describe and represent functions found in a variety of situations with tables, graphs, verbal rules, and equations using variables.

STATISTICS AND PROBABILITY

In order to develop an understanding of **Statistics and Probability**, the mathematics curriculum must include problems which require students to collect, organize, represent, and interpret data; make inferences and predictions; present and evaluate inferences and predictions; present and evaluate arguments based on data analysis; and model situations to determine theoretical and experimental probabilities.

Learning Expectations

Through the investigation of meaningful problems, individually or in cooperative groups while using appropriate technology and employing equity for all, all students in grades 3-5, building upon the K-2 expectations, will be able to:

- construct and use appropriate representations of data through data collections and organizations a variety of displays including line plots, tables, histograms, scale pictographs, and charts;
- interpret data and make convincing arguments that are based on data analysis, including mean, median and mode;
- represent all possible outcomes for an experiment by using a tree diagram;
- determine the probability of a single event based on an experiment with equally likely outcomes;
- formulate and solve problems that involve collecting and analyzing data.

SPATIAL SENSE AND GEOMETRIC CONCEPTS

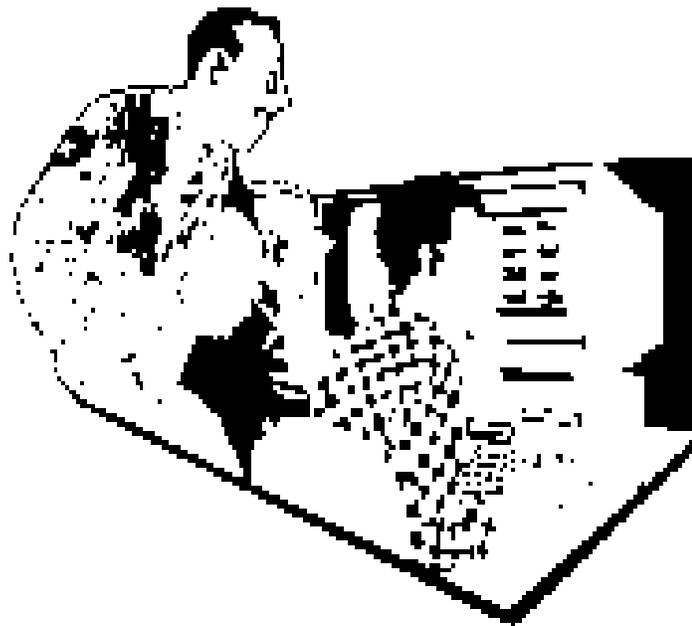
In order to develop **Spatial Sense** and an understanding of **Geometric Concepts**, the mathematics curriculum must include problems which require students to explore geometric properties and relationships and to investigate, model, and analyze one-, two-, and three-dimensional figures.

Learning Expectations

Through the investigation of meaningful problems, individually or in cooperative groups while using appropriate technology, all students in grades 3-5, building upon K-2 expectations, will be able to:

- identify, compare, and accurately represent plane figures using a variety of materials;
- describe and compare properties of polygons such as number of sides, number of vertices, classification of angles and parallel or perpendicular sides;
- apply and explain geometric properties and relationships such as congruency, similarity, and symmetry;
- identify and construct models of three dimensional figures both with and without nets;
- demonstrate transformations using models and pictorial representations;
- apply and compare concepts of area, perimeter, circumference, and volume.

6 - 8



6 - 8

NUMBER SENSE AND NUMBER THEORY

In order to develop **Number Sense** and an understanding of **Number Theory**, the mathematics curriculum must include problems which require students to recognize, represent, and model real numbers and operations verbally, physically, symbolically, and graphically.

Learning Expectations

Through the investigation of meaningful problems, individually or in cooperative groups while using appropriate technology, all students in grades 6-8, building upon K-5 expectations, will be able to:

- represent and use numbers in a variety of equivalent forms, including integers, fractions, decimals, percent, and exponential and scientific notation;
- articulate and model the relationship among decimals, fraction, ratios and percents;
- model integers and their operations;
- model various interpretations, including ratio, of rational numbers

ESTIMATION, MEASUREMENT, AND COMPUTATION

In order to develop understanding of **Estimation, Measurement, and Computation**, the mathematics curriculum should include problems which require students to apply appropriate tools and units of measurement, to develop effective estimation and computation strategies for producing reasonable results, and to calculate using appropriate tools such as mental mathematics, technology, manipulatives, and pencil-and-paper.

Learning Expectations

Through the investigation of meaningful problems, individually or in cooperative groups while using appropriate technology, all students in grades 6-8, building upon K-5 expectations, will be able to:

- compute and estimate with rational numbers including whole numbers, fractions, decimals, and integers;

- explain the concepts underlying computational procedures involving rational numbers, including whole numbers, fractions, decimals, and integers;
- demonstrate understanding of the concepts of perimeter, area, volume, angle measure, capacity, weight and mass, and of any related formulas;
- apply formulas and explain strategies for solving problems related to measurement;
- demonstrate an understanding of rates and other derived and indirect measurements;
- select and apply an appropriate method for computing from among mental arithmetic, paper and pencil, and appropriate technology and evaluate the reasonableness of results.

PATTERNS, FUNCTIONS, AND ALGEBRAIC THINKING

In order to develop **Algebraic Thinking** through an understanding of **Patterns and Functions**, the mathematics curriculum must include problems which require students to describe, extend, analyze, and create a wide variety of patterns and functions using appropriate materials and representations such as tables, graphs, and mathematical notation.

Learning Expectations

Through the investigation of meaningful problems, individually or in cooperative groups while using appropriate technology, all students in grades 6-8, building upon K-5 expectations, will be able to:

- describe, extend, analyze, and create a wide variety of patterns in numbers, shape, and data using a variety of appropriate materials, including manipulatives and technology;
- apply understanding of functional relationships to explain how a change in one quantity results in a change in another;
- use mathematical language and symbols to describe patterns and represent functional relationships;
- identify patterns and functions from statistical data;
- use patterns and functions to represent and solve problems;
- demonstrate an understanding of the concepts of variable, expression, and equation;
- represent situations with tables, graphs, verbal rules, and equations and describe the interrelationships of the representations;

- evaluate algebraic expressions and formulas for given values of the variable using manipulatives, technology and pencil/paper;
- solve linear equations using concrete, informal and formal methods;
- describe and use informal methods to solve linear inequalities and non-linear equations;
- model and solve problems using algebraic methods.

STATISTICS AND PROBABILITY

In order to develop an understanding of **Statistics and Probability**, the mathematics curriculum must include problems which require students to collect, organize, represent, and interpret data; make inferences and predictions; present and evaluate inferences and predictions; present and evaluate arguments based on data analysis; and model situations to determine theoretical and experimental probabilities.

Learning Expectations

Through the investigation of meaningful problems, individually or in cooperative groups while using appropriate technology, and while building upon the K-5 expectations, students in 6-8 should be able to:

- collect, organize, and describe data in order to make conjectures;
- formulate and test hypotheses, draw conclusions, and make convincing arguments that are based on data analysis;
- construct, interpret, and use various displays of data to answer questions and solve problems;
- calculate and use range and measures of central tendency to describe data;
- model situations by devising and carrying out experiments or simulations to determine probabilities;
- make conjectures based on experimental or theoretical probabilities;
- analyze a sample to make inferences about a population;
- construct a sample space to determine the theoretical probability.

SPATIAL SENSE AND GEOMETRIC CONCEPTS

In order to develop **Spatial Sense** and an understanding of **Geometric Concepts**, the mathematics curriculum must include problems which require students to explore geometric properties and relationships and to investigate, model, and analyze one-, two-, and three-dimensional figures.

Learning Expectations

Through the investigation of meaningful problems, individually or in cooperative groups while using appropriate technology, all students in grades 6-8, building upon the K-5 expectations, will be able to:

- demonstrate geometric relationships through the use of manipulatives, graphing calculators, computers, and construction tools such as compass, straight edge and protractor;
- apply and explain effects of basic transformations on geometric figures;
- apply geometric properties and relationships to make conjectures;
- articulate geometric properties and relationships such as congruence, similarity, parallelism, perpendicularity, and symmetry;
- apply geometric concepts to problems in other fields and disciplines;
- represent and solve problems using geometric models.

Informal Glossary of Mathematical Terms from the *Tennessee Mathematics Framework for Grades K-8*

Introduction...

The Informal Glossary of Mathematical Terms from the Tennessee Mathematics Framework for Grades K-8 is a reference to accompany the framework document in order to further describe, clarify, or give examples of selected terms in the framework. One criterion for inclusion in the glossary is that the entry be a form of a word or phrase in the framework document.

The goal for this glossary is to provide definitions to aid teachers with any level of mathematics background to more fully understand the framework document. This glossary is not designed for student use and should not be considered a complete or appropriate list of terms and definitions for K-8 students. Further, it does not replace a mathematical dictionary which should be used to obtain more precise definitions. The definitions given are restricted to only the common mathematical meaning.

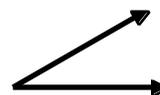
The terms and definitions included in this glossary were produced by the Mathematics Curriculum Framework Committee for Grades K-8.

Informal Glossary of Mathematical Terms
from the
Tennessee Mathematics Framework for Grades K-8

algebraic expressions - numerals and/or variables joined by any combination of the four basic operations (+, -, X, /) and involving any power(s) of the numerals and/or variables; for example: $3 - 8$, 7×4 , $4 + X$, $Y/2$, $N - 2$, $3(4 + 8) - 7$, $Y^2 - 2$

algebraic thinking - thinking skills which are developed by working with problems which require students to describe, extend, analyze, and create a variety of oral, visual, and physical patterns such as ones based on color, shape, number, sounds) from real life and other subjects such as literature and music

angle - two rays which share a common end point

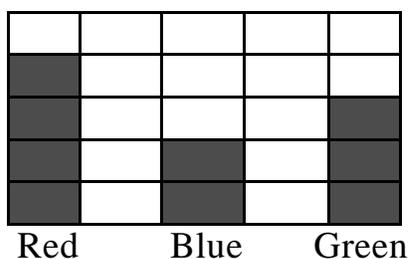


algorithm - a procedure or set of steps for an operation or computation such as long division

area - the number of a given square unit which covers a given region or surface such as the interior of a square

attribute - quality or characteristic

bar graph - a representation of data in which the length of a rectangle or bar is used to represent a numerical amount; a bar graph typically has spaces between the bars; for example



capacity - the number or the amount that a given container can hold expressed in terms of some given unit such as milliliters, cups, liters, quarts, marbles

central tendency - measures of central tendency refer to mean, median, and mode (see mean, median, and mode below)

circumference - the distance around a circle; $\pi \cdot D$ or pi times the diameter

class - a group or set based on common characteristics

compass - a device or tool for drawing circles and arcs

congruent - geometric figures having the same size and shape; all matching or corresponding parts of congruent figures have the same measure

conjecture - a preliminary statement or hypothesis that something is true; the statement may later be confirmed or disproved through observation or testing

corner - a point where lines, edges, or sides meet or converge

derived measurement - (see also indirect measurement) measurement in which measures cannot be calculated directly but must be determined by finding other direct measures; for example: rates such as miles per hour

directional - having to do with the line or course along which an object or figure is moved

equation - two mathematical expressions joined by an equal sign

estimation - an approximation of a measure or calculation based on a strategy such as rounding, front-end estimation, compatible numbers (see also reasonableness of results)

even number - a number that is divisible by 2, such as -4, -2, 0, 2, 4

experimental probability - the actual number of occurrences of a particular (favorable) outcome divided by the total number of trials or outcomes for a particular experiment; as the total number of trials or outcomes increases, the closer the experimental probability comes to the theoretical probability (see theoretical probability)

exponential notation - expressing numbers or algebraic expressions with exponents or powers; for example, expressed in exponential notation $3 \times 10^2 + 4 \times 10^1 + 5$ is 345

expression - a number, variable, or a sum, difference, product, or quotient of numbers and/or variables involving any power of the variable or number;

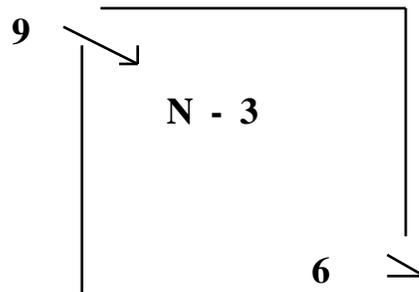
examples: $16 + 4$, $5(3 + 6)$, $3x + 4$ or 5 or $5Y - 3/X$, 7

factor - a number (usually an integer) or algebraic expression that divides (remainder = 0) a given number or expression or that multiplied by another number or expression equals the given number; for example, 1, 2, 3, and 6 are factors of 6

flip - (reflection) the mirror image resulting from flipping or reflecting a point or figure over a given line

function - a relation in which each element of one set (the domain) is paired with one element of a second set (range); functions are sometimes represented by tables or by function machines (see examples below); the graph of a function is one in which a vertical line will intersect the graph exactly once; for example $y = 3x - 4$ defines a function and $y^2 = x$ does not define a function in x . Examples:

N	$N + 4$
3	7
4	8
5	9
7	11
8	12
10	?



histogram - a graph which represents the frequency distribution for a set of data with rectangles having heights to represent the frequencies and widths to represent the data intervals; a histogram looks similar to a bar graph except there are no spaces between the rectangles or bars

hypothesis - an assumption which may be tested experimentally and/or logically

indirect measurement - measurement which can only be determined by applying theories or principles in conjunction with any customary measuring device(s), such as a ruler, scale, meter (see also derived measurement)

inference - a generalization based on statistical data; a prediction based on sample or experimental data

integer - a member of the set of whole numbers and their opposites
{..., -4, -3, -2, -1, 0, 1, 2, 3, 4, ...}

line plot - a representation of data consisting of points joined by line segments

linear equation - an equation in one or two variables with no exponents other than one and with no products of the variables; for example, $y = x + 4$, $x = -4$, and $3x - 4y = 1/2$ are linear equations; in a rectangular coordinate system, the graph of a linear equation is a line (see also nonlinear equation)

mass - the quantity of matter or material in an object; the mass remains the same independent of gravitational force whereas the weight of an object varies; for example, the mass of a rock is the same on Earth, on the Moon, or on Jupiter; however, its weight varies

mean - the average of a set of numerical data; the sum of a set of numerical data divided by the number of items in the set (see also measures of central tendency)

median - the middle number (if there is an odd number of scores or numbers) or the average of the two middle numbers (if there is an even number of scores or numbers) in a set of data arranged in rank order

mode - the most frequent occurring number in a set of data; a set of data may have more than one mode

model - (verb) to show or illustrate a concept or problem by using physical objects with manipulations of these objects; to use simpler or more familiar objects and situations to explain a new concept, to solve a given problem, or to demonstrate understanding of a concept

model - (noun) a representation which uses physical objects or drawings and their actual manipulation to illuminate concepts or problems

multiple - the product of a given whole number and another number, for example, 6 is a multiple of 2, 36 is a multiple of 6, and 0 is a multiple of any number

net - a 2-dimensional representation or model of a 3-dimensional figure or solid which may be obtained by unfolding the figure to show all of its faces

non-linear equation - an equation whose graph is not a line;
ex. $1/x = 7$, $3xy + 4 = y$, or $x^3 = 7$

number relationship - a way in which a given number is expressed in terms of one or more other numbers; for example, $10 = 8 + 2$, $6 \times 6 = 36$; known number relationships can be used to derive others such as $10 - 8 = 2$ or $6 \times 7 + 42$, from the two above

odd - odd numbers are integers which are not divisible by 2; for example, -3, -1, 1, 3, 5

parallel - parallel lines are lines which lie in the same plane but never intersect or meet

percent - a special ratio that compares a number to 100 and uses the % sign; for example: $1/2 = 50\%$ and $2/3 = 66\frac{2}{3}\%$

perimeter - the distance around an object or geometric figure (simple closed curve) expressed as a number of linear units

perpendicular - lines, segments, or rays are perpendicular if they intersect to form a right angle

polygon - a closed curve composed of line segments that meet or intersect at their endpoints or vertices

prime number - a whole number which is equal to or greater than 2 and which has exactly two factors, 1 and itself; for example, 2, 3, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 63, 67, 71, 73, 79, 83, 89, 91, and 97 are the primes less than 100

problem solving - applying or putting together knowledge and skills already learned in new situations or to derive new knowledge and/or solutions

property of operations - one of the characteristics that are always true, for example, the commutative property of addition which means the order of addends is not important: $2 + 3 = 3 + 2$ the associative property of addition means that factors can be grouped in different ways: $[2 + (3 + 5)] = [(2 + 3) + 5]$, and the distributive property means that the product of a number and the sum of two others is equal to the sum of the products of the number and each of the others: $5(7 + 6) = (5)(7) + (5)(6)$; other properties of operations of real numbers include the commutative property of multiplication, the associative property of addition, the zero property of addition, the zero property of multiplication, the additive inverse property, the multiplicative inverse property, and the closure properties for addition, subtraction, and multiplication

protractor - a tool or device for determining the measurement of angles; the unit for measurement in elementary school is generally degrees, however, radians may also be used

rational number - a number that can be expressed as a ratio of two integers a/b with b not equal to zero; for example $3/4$, 7 , $.5$, $.75$, $.333\dots$, 7.5 Pi; the square root of 2, the square root of 3, $.01001000100001\dots$, etc. are not rational numbers

ratio - a comparison of two numbers; notation 2:3; one interpretation of a fraction is that of a ratio, for example, $2/3$ can be interpreted as 2 for every 3 or 2:3

reasonableness of results - acceptability of an estimated or approximate measurement or calculation based on its making sense

relative magnitude of numbers - size in comparison to other numbers; for example, $2 < 3$, $5 > -4$, $6 = 12/2$

sample space - all possible outcomes of an experiment

scientific notation - representation of a number as the product of a number between 1 and 10 and a power of 10; used especially for very small or very large numbers - examples: $6,900,000 = 6.9 \times 10^6$ or $.00069 = 6.9 \times 10^{-4}$

similar - that which has the same shape but not necessarily the same size; figures that are similar have congruent corresponding or matching angles and the corresponding or matching sides are proportional or have equal ratios; for example, all equilateral triangles similar but not congruent

simple chance event - a chance event consisting of a single experimental outcome; for example in rolling a die, there are six simple chance events or outcomes; for example: the event of rolling a 3 or {3} consists of a single outcome and thus is a simple chance event ; on the other hand, the event rolling an even number ({2,4,6}) is consists of three outcomes and thus is not a simple event

simulation - a representation of a situation or problem with a similar but simpler model or a more easily manipulated model in order to determine experimental results

slide - a geometric transformation which involves moving the figure up or down or right or left without changing its orientation; a slide can be thought of as a combination of two flips or reflections about parallel lines

spatial sense - an intuitive feel for one's surroundings and the objects in them (NCTM, 1989); spatial sense involves understanding all types of geometric relationships

statistical data - qualitative or quantitative information collected through experimentation and/or observation for the purpose of analysis, presentation (such as graphic) interpretation, or inference

straight edge - a device for drawing representations of lines, such as, a ruler

symmetry - a property of figures such that for some given dividing line the parts on opposite sides of the line are the same size and shape and have the same relative position from the line

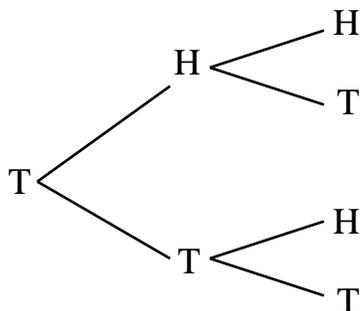
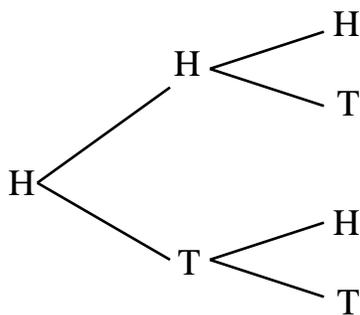
tally - a way of counting or recording which involves making four vertical marks with the fifth a diagonal across; for example,

||||/ || would represent 7

technology - any of the many calculator or computer related tools which can be used for development of mathematical concepts and for problem solving, including but not limited to calculators; graphing calculators; calculator-based laboratories; computers; computer software, such as interactive geometric software, spreadsheets, database software, simulation software, and presentation software; videos, CD's, laser disks

theoretical probability - a numerical measure of the likelihood that an event will occur; the ratio of the number or proportion of successes or favorable outcomes in the sample space to the number of outcomes in the sample space; the expected results (see also experimental probability); for example if a coin is flipped 50 times with 30 heads and 20 tails, the experimental probability of heads is $30/50$ or $.6$; whereas, the theoretical probability for obtaining heads is $25/50$ or $1/2$ or $.5$

tree diagram - a representation of the different possibilities for outcomes of an experiment; example, if a coin is tossed three times, the tree diagram below represents the possible outcomes (HHH, HHT, HTH, HTT, THH, THT, TTH, TTT)



turn - a geometric transformation which involves the rotation of a figure in which every point moves through a given number of degrees around a given point; the composition of two flips over intersecting lines of reflection

variable - a letter or symbol which represents one or more numbers; that is, the value of the variable can 'vary'

verbal rule - a statement in words (written or spoken) which describes the relationship between two sets of numbers as opposed to representing the relationship symbolically

volume -the measure of the amount of space a three dimensional figure takes up or occupies expressed in terms of some cubic unit such as cubic inches, cubic centimeters, etc.

weight - a measure of the mass of an object based on the length it will pull or push down a spring; the weight will vary depending upon the gravitational pull; for example, the weight of a person is different on the Earth from on the Moon (see also mass)

whole number- a member of the set of counting or natural numbers plus 0; {0,1,2,3,4,...}