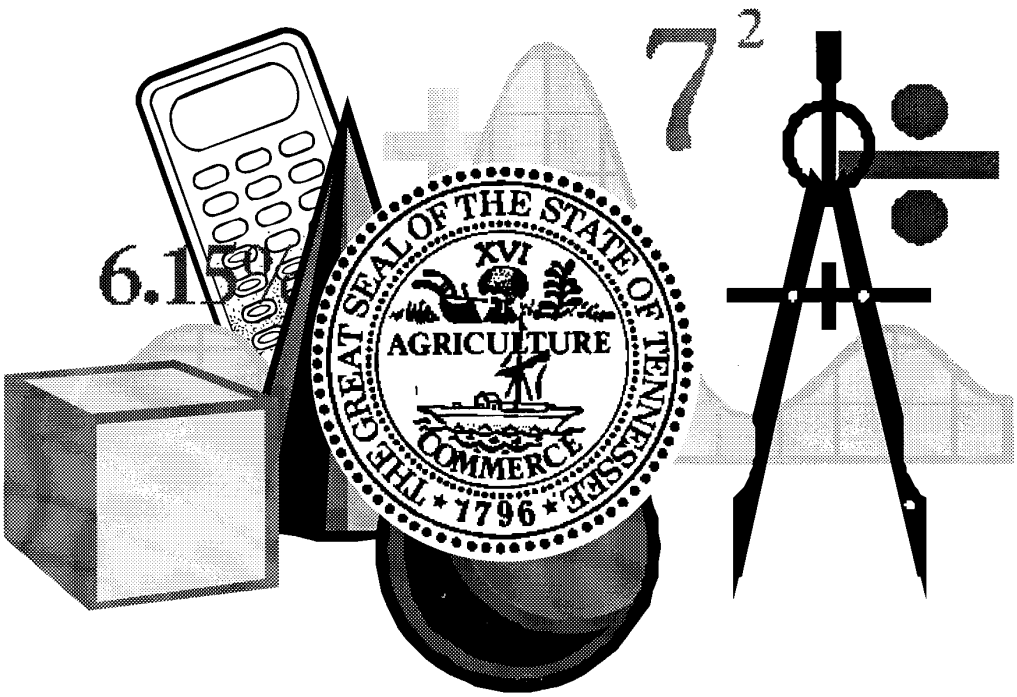
# MATHEMATICS FRAMEWORK

## Grades Nine through Twelve

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Tennessee Mathematics Teachers
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for implementation 1999 - 2000

# MATHEMATICS
# CURRICULUM FRAMEWORK
# GRADES 9 - 12

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# PREFACE

Consistent with the six-year curriculum development and textbook adoption cycle of Rules, Regulations, and Minimum Standards of the Tennessee State Board of Education, the 9-12 Mathematics Framework has been developed by a statewide committee of Mathematics educators. The framework includes the **Content Standards** and **Learning Expectations** necessary to insure 9-12 Tennessee students develop the mathematics skills needed to succeed in school, in the workplace, and in their lives.

The foundation of the framework is the four Process Standards: *Mathematics as Problem Solving, Mathematics as Communication, Mathematics as Reasoning, and Mathematical Connections*. These four Process Standards are intended to be incorporated into each of the five Content Standards: *Number Sense and Number Theory; Estimation, Measurement, and Computation; Patterns, Functions, and Algebraic Thinking; Statistics and Probability; and Spatial Sense and Geometric Concepts*. The Learning Expectations identify the essential core of learning for each mathematics course.

This document contains the minimum expectations and shall be the basis for planning instructional programs at the local level. School systems have the flexibility to broaden their programs of study with regard to the depth and breadth of material to be covered at each grade level. The sequencing of objectives and the development of curriculum guides and lesson plans are the responsibility of system level instructional leaders.

## 9 - 12 MATHEMATICS CURRICULUM FRAMEWORK COMMITTEE

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# TABLE OF CONTENTS

# PHILOSOPHY

In its July 31, 1992, "Mathematics Policy," the Tennessee State Board of Education states: "All students must have access to a rich curriculum emphasizing mathematical thinking and problem solving in order to ensure a mathematically literate work force and to promote equal opportunity for all citizens." The document sets forth the following goals for all students: "that they (1) learn to value mathematics, (2) become confident in their ability to do mathematics, (3) become mathematical problems solvers, (4) learn to communicate mathematically, and (5) learn to reason mathematically."

The following year the Tennessee State Board of Education articulated the "High School Policy: A New Vision for Tennessee High Schools." (September 17, 1993) Based upon current educational research, these recommendations for the mathematics curriculum were made:

Students will:

a. Read, write, and orally communicate mathematical concepts.
b. Use various methods, including mental math, estimating,
c. Organize, analyze, depict, and interpret data to make decisions and predictions related to real-world situations.
d. Use appropriate tools, such as measuring instruments, calculators, and computers, to solve problems.
e. Solve theoretical and practical problems using essential concepts of algebra, geometry, probability, and statistics.
f. Understand the relationship between mathematics, the sciences, technology, and society ("High School Policy: A New Vision for Tennessee High Schools"; 1993).

This committee has strived to outline a Secondary Mathematics Curriculum Framework that is in accordance with these. This Framework outlines several course sequences which set high, but not unreasonable, expectations that are designed to help prepare all students for citizenship, for work, and for higher education.

In the past, pencil-and-paper algorithms have been the primary tools of mathematics that were taught in school. Such a focus is inadequate for today's world. Because calculators and computers are widely accessible, the nature of the problems of mathematics and the methods used to

investigate them have changed tremendously. Indeed, technology is pervasive in government, industry, and business. Students now need to know how to use a variety of tools, including technology and mental calculations as well as pencil and paper. The proposed Framework strongly supports the implementation of technology throughout the mathematics curriculum.

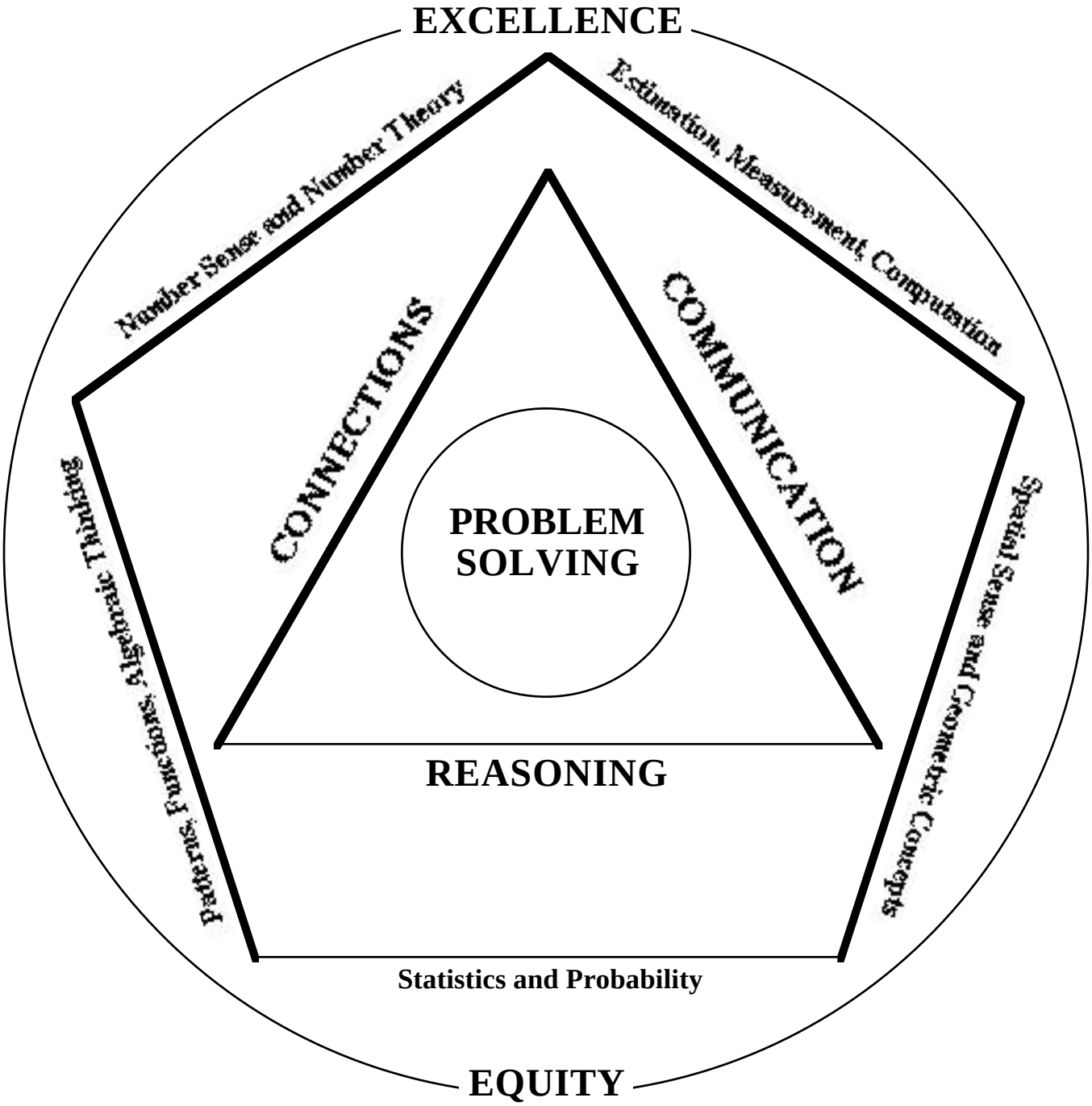
The organization of this Framework is a two-level hierarchy of expectations, all of which are intended to advance the goals previously stated. The first level of expectations consists of the four Process Standards: (1) problem solving, (2) reasoning, (3) communication, and (4) connections. The second level is made up of the five Content Standards. The four Process Standards permeate the Content Standards; that is, we acknowledge that teaching any particular content well involves creating an environment in which students are engaged in significant problem solving, in sense-making, in mathematical discourse, and in recognizing inter-relationships.

The issue of equity, particularly in mathematics education, is crucial. Mathematics is a filter for employment. That is, lack of mathematical background limits job opportunities. Therefore, it is vital that all students, especially females and minorities who have traditionally been underrepresented in mathematics-intensive fields, be strongly supported in mathematics education. In order to promote equity, this Framework is designed so that all students who meet the three mathematics credit graduation requirement will have studied these five content standards: Number Sense and Number Theory; Estimation, Measurement, and Computation; Patterns, Functions, and Algebraic Thinking; Statistics and Probability; and Spatial Sense and Geometric Concepts.

As suggested by the Tennessee State Board of Education "Mathematics Policy" (1992), the Framework includes the option of a three-year integrated curriculum. An integrated curriculum would allow opportunities for learning that reflect the connections among the various strands of mathematics. For instance, students could experience the usefulness of taking a geometric approach to a probability problem. It also provides for students to visit the same mathematical concepts from several perspectives. However, moving to such a curriculum will require time for professional development, for exchange of information, and for students to complete currently offered courses of study they have already begun. Courses in the traditional sequence also include an increased emphasis in the four process standards. School districts should carefully weigh the merits of each option and offer that which they think is most effective for their students.

Tennessee joins other states in basing the State Curriculum Framework on systemic reform. In fact, our task was made less daunting by the prior work of committees in other states, such as Delaware, Georgia, and Virginia, has made. It is the belief of this committee that implementation of the recommendations stated in this document will enable Tennessee to improve the mathematics education of its students. This committee understands that the changes delineated here are not merely adjustments of the former curriculum. Instead, this document represents systemic change in mathematics education that will require extensive staff development and necessitate new forms of assessment that reflect the emphasis on higher order thinking.

# MATHEMATICAL EMPOWERMENT OF STUDENTS




**Empowering all students mathematically is the focus of this document.**

# HISTORY

The Basic Skills First curriculum initiatives that began in the late 1970's involved elementary and middle school teachers and mathematics curriculum specialists in identifying by grade level, K-8, the skills and concepts that should be taught. This effort resulted in the first statewide mathematics curriculum document which, with later revisions, became the Blue Book. Key skills at the 6th-8th grade level were identified as minimum proficiency skills and policy was adopted which required students to pass a test based on these skills in order to receive a high school diploma. The list of proficiency skills has been revised several times since the late 1970's. In 1993, another revision occurred which resulted in the TCAP Competency Test. The Competency Test is based on 8th grade skills as identified in the state framework. Passing it remains a graduation requirement in order for students to receive a high school diploma.

In 1984 the Comprehensive Education Reform Act (CERA) was passed by the Tennessee Legislature with the stipulation that within five years instructional programs should be improved in a measurable way. Mathematics, as described in Academic Preparation for College: What Students Need to Know and Be Able to Do (the Green Book), was included as a subject to show measurable improvement. In 1986 the Tennessee State-Wide School-College Collaborative for Education Excellence (the Collaborative) was organized and included several Task Forces, including one for Mathematics. The Mathematics Task Force looked at the existing State Mathematics curriculum documents and produced the two documents, Mathematics Framework (K-8) and Mathematics Curriculum Guide (9-12). These documents were correlated to the Green Book as required by CERA.

In 1989 the NCTM Curriculum and Evaluation Standards for School Mathematics was published. A cadre of mathematics educators from across the state, including K-16 representation, was assembled to receive training for leaders offered by NCTM and to then conduct awareness workshops across the state regarding the Standards. In addition, this cadre reviewed the K-8 and 9-12 frameworks in light of the Standards and made some revisions of those documents in 1991. Also in 1991 Math for Technology I was added to the state curriculum for 9-12 and the Professional Standards for Teaching Mathematics was published by NCTM.

In 1992 the Tennessee Legislature passed the Education Improvement Act based on the 21st Century Challenge Plan of 1990. As a result, several initiatives began. The Tennessee Comprehensive Curriculum Guide, Grades K-8 was assembled. It included skills and concepts for Language Arts, Mathematics, Science, and Social Studies for these grades in one document that became known as the Blue Book. Also, the State Board of Education adopted a Mathematics Policy endorsing the NCTM Curriculum and Evaluation Standards for School Mathematics and several mathematics professional groups and individuals across the state began efforts to review the existing frameworks and guides in light of the Standards. One of these groups was formed by the Systemic Initiative Steering Committee and received funding from the Eisenhower Math/Science Consortium at Appalachia Educational Laboratory to prepare a companion document to the frameworks that would connect these two frameworks with the NCTM standards documents. The document, Mathematics for All Tennessee Students, was written in 1994 with the goal of making the frameworks more useful to teachers and more compatible with the Standards.

In 1993 Math for Technology II was approved as a high school course and was certified by governing bodies as meeting the Algebra I admission requirement for state universities.

In 1994 the high school policy requiring the graduation requirement of 3 years of mathematics including Algebra I or the equivalent, Math for Technology II, was implemented.

In 1995 preparation for instituting End of Course Assessments in high school mathematics courses began. Task forces of teachers from across the state were convened to write a bank of items in all the existing mathematics courses included in the existing curriculum framework. The items were submitted to CTB/McGraw Hill to be used to construct subject area tests for Pre-Algebra, Math for Technology I, Algebra I, Geometry, and Algebra II. Preparation for tests for other mathematics courses was delayed.

In the spring of 1996 a team of mathematics teachers and curriculum specialists representing K-16 and the entire state was convened to rewrite the K-8 state frameworks based on the Standards and current practice. The revision was then reviewed by mathematics educators statewide. This effort resulted in the production of the *K-8 Mathematics Framework* which identifies the Process Standards, Content Standards, and Learning Expectations that should guide state school systems in making the changes needed to improve the learning of mathematics by students across the state.

Building upon the expectations of the *K-8 Mathematics Framework*, a team representing high school and university mathematics educators began revising the *Secondary Mathematics Framework* in February of 1997.

# *Process Standards*

# MATHEMATICS AS PROBLEM SOLVING

The study of mathematics must emphasize Problem Solving opportunities which require various approaches to investigate, understand, and apply mathematical concepts.

The development of each learner's ability to solve problems is essential if he or she is to be a productive citizen. We strongly endorse the first recommendation of An Agenda for Action (NCTM, 1980): "Problem solving must be the focus of school mathematics." To develop such abilities, students need to work on problems that may take hours, days, and even weeks to solve. Some may be relatively simple exercises to be accomplished independently; some should involve small groups or an entire class working cooperatively; and some problems should also be open-ended with no single right answer.

"Mathematics as Problem Solving" emphasizes the learners' use of a broad base of strategies to:

Investigate and understand mathematical content
Recognize and formulate problems from within and outside of mathematics
Use mathematical modeling and appropriate technology to solve a wide variety of problems, including real-world problems.
Generalize solutions and strategies, applying them to new problems.
Increase confidence in their ability to use mathematics meaningfully and to become independent problem solvers.

## MATHEMATICS AS COMMUNICATION

The study of mathematics must emphasize Communication by requiring opportunities to explain, conjecture, summarize, and defend one's ideas orally, in writing, and through the use of technology.

The development of a learner's power to think mathematically involves learning the signs, symbols, and terms of mathematics. This is best accomplished in problem situations in which students have an opportunity to read, write, and discuss ideas in which the use of the language of mathematics becomes natural. As students communicate their ideas, they learn to clarify, refine, and consolidate their answers.

"Mathematics as Communication" focuses on the learners' development of using language and symbols to:

- Reflect and clarify thinking about mathematical ideas and situations.
- Express mathematical ideas and relationships, orally, in writing, and with physical material, pictures, and diagrams
- Understand and value the role of mathematical notation.
- Realize that representing, discussing, listening, writing, and reading mathematics are vital aspect of mathematics study and use.
- Use mathematical notation to formulate generalizations.

## MATHEMATICS AS REASONING

The study of mathematics must emphasize Reasoning which requires critical thinking, logical argument, and justification of solutions, of thought processes, and of conjectures.

Making conjectures, gathering evidence, and building an argument to support such notions are fundamental to doing mathematics. In fact, a demonstration of good reasoning should be rewarded even more than the learner's ability to find correct answers.

"Mathematics as Reasoning" concentrates on leading the learners to:

- Make and test mathematical conjectures.
- Make, follow, and judge the value of mathematical arguments
- Draw logical conclusions.
- Justify solution-finding processes and answers.

## MATHEMATICAL CONNECTIONS

The study of mathematics must emphasize making Connections among the various topics within mathematics, between mathematics and other disciplines, and between mathematics and "real world" situations.

The mathematics curriculum is often viewed as consisting of several discrete stands; so topics tend to be taught in isolation. Unless the learners connect ideas both among and between areas of mathematics, they learn isolated skills rather than develop the ability to recognize general principles and procedures relevant to several areas. Connecting conceptual understanding to procedures will enable learners to apply, recreate, and invent new procedures when needed. Failure to connect conceptual understanding to procedures results in a view of mathematics as an arbitrary set of rules. Learners should have many opportunities to observe and work with the interaction of mathematics with other subjects and with everyday society. Problems become meaningful when they relate to the learners' experiences. Mathematics must be integrated into contexts that give its symbols and processes practical meaning. The school environment is rich with opportunities to use mathematics in other subject areas as well as other subject area content in mathematics.

"Mathematical Connections" concentrate on enabling the learners to:

- Appreciate mathematics as an integrated whole, linking conceptual and procedural knowledge within the discipline and relating multiple representations of concepts or procedures to one another.
- Apply mathematical thinking and modeling to solve substantial problems that arise in other disciplines and curriculum areas, such as art, business, music, psychology, industrial arts, computer technology, social studies, and sciences, such as biology, chemistry, and physics.
- Use, recognize, and value the varied roles of mathematics in their lives, cultures, and society.

# 9-12 MATHEMATICS COURSE LISTING

| Course | Course Code |
|---|---|
| Competency Mathematics | 3101 |
| Foundations I | 3130 |
| Foundations II | 3131 |
| Algebra I | 3102 |
| Geometry | 3108 |
| Algebra II | 3103 |
| Integrated Mathematics I | 3132 |
| Integrated Mathematics II | 3133 |
| Integrated Mathematics III | 3134 |
| Advanced Algebra and Trigonometry | 3124 |
| Discrete Mathematics with Statistics and Probability | 3135 |
| PreCalculus | 3126 |
| Statistics | 3136 |
| Calculus | 3113 |

Vocational courses which receive mathematics credit are listed below. The frameworks for these courses may be found in the State Department of Education's ***Technology Preparation Framework***.

| | |
|---|---|
| Mathematics for Technology I | 3114 |
| Mathematics for Technology II | 3123 |

Advanced placement mathematics courses are listed below. Descriptions for these courses may be found in the ***Advanced Placement Course Descriptions*** provided by The College Board.

| | |
|---|---|
| Calculus AB | 3127 |
| Calculus BC | 3128 |
| Statistics | 3129 |

# Traditional Mathematics Course Sequence

**Students may receive mathematics credit for only ONE of the following: Algebra I, Mathematics for Technology II, or Integrated Mathematics I. It is strongly recommended that students who begin in the Traditional Program (Alg I, Geom, Alg II) not move to the Integrated program. Transfer students may be an exception to this recommendation.**

| | | | |
|---|---|---|---|
| **Level 0** | | Competency Mathematics (no mathematics credit) | **The course at this level is a review of the mathematics content of the 1996 K-8 Mathematics Framework** |
| **Level 1** | Mathematics for Technology I | Foundations I<br><br>Foundations II | **This level is for students not ready to enter Alg I or its equivalent. Courses at this level are not required, however, students can receive credit for at most TWO of these courses.** |
| **Level 2** | Mathematics for Technology II | Algebra I | **At least one of the three mathematics credits required for graduation must be from Level 2 or beyond.** |
| **Level 3** | | Algebra II<br>or<br>*Geometry<br><br>*** Geometry is considered an advanced mathematics course** | **The University Path mathematics requirements are Alg I, Alg II, and an advanced mathematics course.** |
| **Level 4** | | Algebra II<br>or<br>*Geometry<br><br>*** Geometry is considered an advanced mathematics course** | **The University Path mathematics requirements are Alg I, Alg II, and an advanced mathematics course.** |
| **Level 5** | Advanced Algebra & Trigonometry<br><br>Statistics | Discrete Mathematics With Statistics & Probability<br><br>PreCalculus<br><br>Calculus | **Courses at this level provide study in advanced mathematics.** |

## Integrated Mathematics Course Sequences

**Students may receive mathematics credit for only ONE of the following: Algebra I, Mathematics for Technology II, or Integrated Mathematics I. It is strongly recommended that students who begin in the Integrated Program (Integrated Mathematics I, II, & III) not move to the Traditional Program. Transfer students may be an exception to this recommendation.**

| | | | |
|---|---|---|---|
| **Level 0** | | Competency Mathematics (no mathematics credit) | **The course at this level is a review of the mathematics content of the 1996 K-8 Mathematics Framework** |
| **Level 1** | Mathematics for Technology I | Foundations I<br><br>Foundations II | **This level is for students not ready to enter Alg I or its equivalent. Courses at this level are not required, however, students can receive credit for at most TWO of these courses.** |
| **Level 2** | Mathematics for Technology II | Integrated Mathematics I | **At least one of the three mathematics credits required for graduation must be from Level 2 or beyond.** |
| **Level 3** | | Integrated Mathematics II | **Integrated Mathematics I, II, and III meet the University Path requirements for mathematics** |
| **Level 4** | | Integrated Mathematics III | **Integrated Mathematics I, II, and III meet the University Path requirements for mathematics** |
| **Level 5** | Advanced Algebra & Trigonometry<br><br>Statistics | Discrete Mathematics With Statistics & Probability<br><br>PreCalculus<br><br>Calculus | **Courses at this level provide study in advanced mathematics.** |

# *Content Standards and Learning Expectations by Course*

# COMPETENCY MATHEMATICS

(This course will not serve as one of the required mathematics courses for graduation)

**1 Unit**

**Course Number:** 3101

**Level:** Level 0 course

**Recommended Prerequisite:** None

**Emphasis:** Through the investigation of meaningful problems individually or in cooperative groups, while using appropriate technology, students will meet the learning expectations listed below.

## Learning Expectations:

### Number Sense and Number Theory

- represent and use numbers in a variety of equivalent forms, including integers, fractions, decimals, percent, and exponential and scientific notation;
- articulate and model the relationship among decimals, fraction, ratios, and percents;
- model integers and their operations;
- model various interpretations, including ratio, of rational numbers;
- use a number line to represent and compare rational numbers.

### Estimation, Measurement, and Computation

- compute and estimate with rational numbers including whole numbers, fractions, decimals, and integers;
- explain the concepts underlying computational procedures involving rational numbers, including whole numbers, fractions, decimals, and integers;
- demonstrate understanding of the concepts of perimeter, area, volume, angle measure, capacity, weight and mass, and of any related formulas;
- select and apply an appropriate method for computing from among mental arithmetic, paper and pencil, and appropriate technology; and evaluate the reasonableness of results;

- apply rational number concepts and operations in real-world problem solving;
- interpret and compute complex numerical expressions involving addition, subtraction, multiplication, division, exponents, and grouping symbols.

## Patterns, Functions, and Algebraic Thinking

- describe, extend, analyze, and create a wide variety of patterns in numbers, shapes, and data using a variety of appropriate materials, including manipulatives and technology;
- apply understanding of functional relationships to explain how a change in one quantity results in a change in another;
- demonstrate an understanding of the concepts of variable, expression, and equation;
- represent situations with tables, graphs, verbal rules, and equations; and describe the interrelationships of the representations;
- solve linear equations using concrete, informal and formal methods.

## Statistics and Probability

- collect, organize, and describe data in order to make conjectures;
- construct, interpret, and use various displays of data to answer questions and solve problems;
- calculate and use range and measures of central tendency to describe data;
- model situations by devising and carrying out experiments or simulations to determine probabilities;
- interpret graphs, tables, scales, and charts.

## Spatial Sense and Geometric Concepts

- apply geometric properties and relationships to make conjectures;
- articulate and apply geometric properties and relationships such as congruence and symmetry;
- apply geometric concepts to problems in other fields and disciplines;
- represent and solve problems using geometric models;
- communicate position using spatial sense with two-dimensional coordinate systems.

# FOUNDATIONS I
## 1 Unit

**Course Number:** 3130

**Level:** Level 1 course

**Recommended Prerequisite:** None

**Emphasis:** Through the investigation of meaningful problems individually or in cooperative groups, while using appropriate technology, students will meet the learning expectations listed below.

## Learning Expectations:

### Number Sense and Number Theory

- demonstrate an understanding of the subsets, elements, properties, and operations of the rational number system;
- connect physical, graphical, verbal, and symbolic representations of rational numbers;
- informally describe and model the concept of inverse (e.g., opposites, reciprocals) in real life problem situations;
- apply number theory concepts (e.g., primes, factors, divisibility and multiples) in mathematical problem situations;
- use rational numbers to represent real-world applications ( e.g., probability, proportionality);
- use mathematical notations appropriately;
- demonstrate an understanding of percent in solving real-world problems;
- demonstrate an understanding of pi.

### Estimation, Measurement, and Computation

- select and apply an appropriate method (i.e., mental arithmetic, paper and pencil, or technology) for computing with rational numbers, and evaluate the reasonableness of results;
- communicate the concepts and strategies being used in estimation, measurement, and computation;
- use concepts of length, area, and volume to estimate and solve real-world problems (attention given to parallelograms, triangles, right rectangular prisms, circles, right cylinders);

- apply measurement concepts and relationships in geometric problem-solving situations;
- use estimation to make predictions and determine reasonableness of results;
- choose appropriate techniques and tools to measure quantities in order to meet specifications for precision and accuracy.

## Patterns, Functions, and Algebraic Thinking

- recognize, extend, and create geometric, spatial, and numerical patterns;
- solve problems in number theory, geometry, probability and statistics, and measurement and estimation using algebraic thinking;
- communicate the meaning of variables in algebraic expressions and equations;
- apply the concept of variable in simplifying algebraic expressions and solving equations;
- interpret graphs that depict real-world phenomena;
- model real-world phenomena using graphs.

## Statistics and Probability

- interpret a set of data using the appropriate measure of central tendency (mean, median, mode);
- choose, construct, and analyze appropriate graphical representations for a data set including pie charts, histograms, stem and leaf plots, and scatterplots;
- apply appropriate technology in data collection and analysis;
- use simulations to estimate probability.

## Spatial Sense and Geometric Concepts

- apply geometric properties, formulas, and relationships to solve real-world problems;
- use a number line to communicate position using spatial sense and to model operations;
- communicate position using spatial sense with two-dimensional coordinate systems;
- demonstrate an understanding of the properties and construction of geometric figures, including angles, parallel lines, perpendicular lines, triangles, circles, and quadrilaterals.

# FOUNDATIONS II
## 1 Unit

**Course Number:** 3131

**Level:** Level 1 Course

**Recommended Prerequisite:** None

**Emphasis:** Through the investigation of meaningful problems individually or in cooperative groups, while using appropriate technology, students will meet the learning expectations listed below.

**Learning Expectations:**

### Number Sense and Number Theory

- demonstrate an understanding of the subsets, elements, properties, and operations of the real number system;
- demonstrate an understanding of the relative size of rational and irrational numbers;
- connect physical, graphical, verbal, and symbolic representations of real numbers;
- informally describe and model, the concept of inverse (e.g., opposites, reciprocals, and squares and square roots);
- demonstrate an understanding of division involving zero;
- describe, model, and apply inverse operations;
- apply number theory concepts (e.g., primes, factors, divisibility and multiples) in mathematical problem situations;
- connect physical, graphical, verbal, and symbolic representations of absolute value;
- use real numbers to represent real-world applications (e.g., rate of change, probability, and proportionality);
- use mathematical notations appropriately.

### Estimation, Measurement, and Computation

- select and apply an appropriate method (i.e., mental arithmetic, paper and pencil, or technology) for computing with real numbers, and evaluate the reasonableness of results;
- communicate the concepts and strategies being used in estimation, measurement, and computation;

- perform operations on simple algebraic expressions, and informally justify the procedures chosen;
- use concepts of length and area, including surface area and volume, to estimate and solve real-world problems (e.g., parallelograms, triangles, right rectangular prisms, circles, right cylinders, spheres, and pyramids);
- apply measurement concepts and relationships in algebraic and geometric problem-solving situations;
- use estimation to make predictions and determine reasonableness of results;
- choose appropriate techniques and tools to measure quantities in order to meet specifications for precision and accuracy;
- demonstrate an understanding of rates and other derived and indirect measurements (e.g., velocity, miles per hr, rpm, cost per unit).

## Patterns, Functions, and Algebraic Thinking

- recognize, extend, and create geometric, spatial, and numerical patterns;
- analyze mathematical patterns related to algebra and geometry in real-world problem solving;
- solve problems in number theory, geometry, probability and statistics, and measurement and estimation using algebraic thinking and symbolism (attention given to solving linear equations);
- communicate the meaning of variables in algebraic expressions, equations, and inequalities;
- interpret the results of algebraic procedures;
- apply the concept of variable in simplifying algebraic expressions, solving equations, and solving inequalities;
- interpret graphs that depict real-world phenomena;
- model real-world phenomena using graphs.

## Statistics and Probability

- interpret a set of data using the appropriate measure of central tendency (mean, median, mode) and the appropriate measure of dispersion ( e.g., quartiles, range);
- choose, construct, and analyze appropriate graphical representations for a data set including pie charts, histograms, stem and leaf plots, scatterplots, box and whisker plots;
- apply appropriate technology in data collection and analysis;
- apply theoretical and experimental probability to analyze the likelihood of an event;
- use simulations to estimate probability;
- analyze the validity of statistical conclusions and the use, misuse, and abuse of data;

- apply counting principles of permutations and combinations using appropriate technology.

## Spatial Sense and Geometric Concepts

- analyze relationships among corresponding parts of similar or congruent geometric figures;
- apply geometric properties, formulas, and relationships to solve real-world problems;
- use inductive reasoning to make conjectures;
- communicate position using spatial sense with two-dimensional coordinate systems;
- demonstrate an understanding of transformations of geometric figures;
- apply The Pythagorean Theorem in problem solving;
- name, analyze, and describe the properties of various polygons.

# ALGEBRA I
## 1 Unit

**Course Number:** 3102

**Level:** Level 2 course

**Recommended Prerequisite:** Successful completion of 8th grade mathematics, Foundations I, and/or Foundations II.

**Emphasis:** Through the investigation of meaningful problems individually or in cooperative groups, while using appropriate technology, students will meet the learning expectations listed below.

**Learning Expectations:**

### Number Sense and Number Theory

- demonstrate an understanding of the subsets, elements, properties, and operations of the real number system;
- demonstrate an understanding of the relative size of rational and irrational numbers;
- articulate, model, and apply the concept of inverse (e.g., opposites, reciprocals, and powers and roots);
- describe, model, and apply inverse operations;
- apply number theory concepts (e.g., primes, factors, divisibility and multiples) in mathematical problem solving;
- connect physical, graphical, verbal, and symbolic representations of absolute value;
- use real numbers to represent real-world applications (e.g., slope, rate of change, probability, and proportionality);
- use a variety of notations appropriately (e.g. exponential, functional, square root).

### Estimation, Measurement, and Computation

- select and apply an appropriate method (i.e., mental arithmetic, paper and pencil, or technology) for computing with real numbers, and evaluate the reasonableness of results;
- communicate the concepts and strategies being used in estimation, measurement, and computation;

- perform operations on algebraic expressions and informally justify the procedures chosen;
- use concepts of length, area, and volume to estimate and solve real-world problems;
- apply measurement concepts and relationships in algebraic and geometric problem-solving situations;
- demonstrate an understanding of rates and other derived and indirect measurements (e.g., velocity, miles per hour, revolutions per minute, cost per unit);
- use matrices with appropriate technology in real-world problem solving;
- use estimation to make predictions and test reasonableness of results.

## Patterns, Functions, and Algebraic Thinking

- recognize, extend, and create geometric, spatial, and numerical patterns;
- analyze mathematical patterns related to algebra and geometry in real-world problem solving;
- use algebraic thinking to generalize a pattern by expressing the pattern in functional notation;
- solve linear systems using a variety of techniques, including matrices;
- solve problems in number theory, geometry, probability and statistics, and measurement and estimation using algebraic thinking and symbolism;
- communicate the meaning of variables in algebraic expressions, equations, and inequalities;
- identify and represent a variety of functions;
- apply the concept of rate of change;
- analyze graphs to describe the behavior of functions;
- interpret results of algebraic procedures;
- apply the concept of variable in simplifying algebraic expressions, solving equations, and solving inequalities;
- interpret graphs that depict real-world phenomena;
- model real-world phenomena using functions and graphs;
- articulate and apply algebraic properties in symbolic manipulation;
- identify relationships which can and which cannot be represented by a function;
- graph inequalities and interpret graphs of inequalities;
- describe the domain and range of functions and articulate restrictions imposed either by the operations or by the real-life situations which the functions represent;
- describe the transformation of the graph that occurs when coefficients and/or constants of the corresponding linear equations are changed.

## Statistics and Probability

- collect, represent, and describe linear and nonlinear data sets developed from the real world;
- make predictions from a linear data set using a line of best fit;
- interpret a set of data using the appropriate measure of central tendency;
- choose, construct, and analyze appropriate graphical representations for a data set;
- apply the Law of Large Numbers and the concept of randomness in sampling;
- apply counting principles of permutations and combinations using appropriate technology.

## Spatial Sense and Geometric Concepts

- apply geometric properties, formulas, and relationships to solve real-world problems;
- use inductive reasoning to make conjectures;
- apply right triangle relationships including the Pythagorean Theorem and the distance formula.

# GEOMETRY
## 1 Unit

**Course Number:** 3108

**Level:** Level 3 or 4 course

**Recommended Prerequisite:** Algebra I or its equivalent.

**Emphasis:** Through the investigation of meaningful problems individually or in cooperative groups, while using appropriate technology, students will meet the learning expectations listed below.

## Learning Expectations:

### Number Sense and Number Theory

- demonstrate an understanding of the relative size of rational and irrational numbers.

### Estimation, Measurement, and Computation

- use concepts of length, area, and volume to estimate and solve real-world problems;
- apply measurement concepts and relationships in algebraic and geometric problem-solving situations;
- choose appropriate techniques and tools to measure quantities in order to meet specifications for precision, accuracy, and tolerance.

### Patterns, Functions, and Algebraic Thinking

- recognize, extend, and create geometric, spatial, and numerical patterns;
- analyze mathematical patterns related to algebra and geometry in real-world problem solving;
- solve problems in number theory, geometry, probability and statistics, and measurement and estimation using algebraic thinking and symbolism;
- apply coordinate geometry to analyze and solve problems.

## Statistics and Probability

- apply geometric representations to calculate theoretical probability.

## Spatial Sense and Geometric Concepts

- analyze relationships among corresponding parts of similar or congruent geometric figures;
- apply geometric properties of solids, polygons, and circles to solve real-world problems;
- justify conclusions using deductive reasoning;
- use inductive reasoning to make conjectures;
- communicate position using spatial sense with two- and three-dimensional coordinate systems;
- demonstrate an understanding of transformations of geometric figures (i.e., translations, rotations, dilations, and reflections);
- apply right triangle relationships including the Pythagorean Theorem, the distance formula, and trigonometric ratios;
- describe geometric objects and recognize minimal conditions necessary to define the geometric objects;
- apply reflexive, transitive, and symmetric properties when appropriate;
- demonstrate understanding of geometric properties of congruence, similarity, perpendicularity, and parallelism;
- recognize and articulate relationships among families of geometric figures (e.g., quadrilaterals, prisms);
- use indirect and deductive reasoning to establish the truth of a statement.

# ALGEBRA II
## 1 Unit

**Course Number:** 3103

**Level:** Level 3 or 4 course

**Recommended Prerequisite:** Algebra I or its equivalent.

**Emphasis:** Through the investigation of meaningful problems individually or in cooperative groups, while using appropriate technology, students will meet the learning expectations listed below.

## Learning Expectations:

### Number Sense and Number Theory

- demonstrate an understanding of the subsets, elements, properties, and operations of the complex number system;
- connect physical, graphical, verbal, and symbolic representations of real numbers;
- articulate, model, and apply the concept of inverse (e.g., opposites, reciprocals, and powers and roots);
- describe, model, and apply inverse operations;
- connect physical, graphical, verbal, and symbolic representations of absolute value;
- use a variety of notations appropriately (e.g., logarithmic, factorial, sigma, delta, radical).

### Estimation, Measurement, and Computation

- use estimation to make predictions and determine the reasonableness of results;
- select and apply an appropriate method (i.e., mental arithmetic, paper and pencil, or technology) for computing with real numbers, and evaluate the reasonableness of results;
- apply measurement concepts and relationships in algebraic and geometric problem-solving situations;
- use matrices in real-world problem solving;
- perform operations on algebraic expressions and informally justify the procedures chosen.

## Patterns, Functions, and Algebraic Thinking

- analyze mathematical patterns related to algebra and geometry in real-world problem solving;
- use algebraic thinking to generalize a pattern by expressing the pattern in functional notation;
- solve linear systems using a variety of techniques, including matrices;
- communicate the meaning of variables in algebraic expressions, equations, and inequalities;
- apply the concept of rate of change;
- identify and represent a variety of functions;
- analyze graphs to describe the behavior of functions;
- identify and describe the characteristics of families of functions;
- articulate the results of varying parameters of a parent function;
- demonstrate understanding of the behavior of quadratic, exponential, periodic, and logarithmic functions and their graphs;
- interpret results of algebraic procedures;
- apply the concept of variable in simplifying algebraic expressions, solving equations, and solving inequalities;
- interpret graphs that depict real-world phenomena;
- model real-world phenomena using functions and graphs;
- describe the domain and range of functions and articulate restrictions imposed either by the operations or by the real-life situations which the functions represent;
- use linear programming to solve real-world problems;

## Statistics and Probability

- apply the Law of Large Numbers and the concept of randomness in sampling;
- apply appropriate technology in data collection and analysis;
- apply counting principles of permutations and combinations using appropriate technology;
- apply theoretical and experimental probability to analyze the likelihood of an event;
- collect, represent, and describe linear and nonlinear data sets developed from the real world;
- make predictions from a data set using curve fitting with appropriate technology;
- make inferences about a data set using appropriate measures of central tendency and dispersion;
- describe and apply the normal distribution and its properties;
- identify mutually exclusive and non-mutually exclusive events;
- analyze the probability of dependent events and of independent events;
- use simulations to estimate probability;

- choose, construct, and analyze appropriate graphical representations for a data set;
- analyze the validity of statistical conclusions and the use, misuse, and abuse of data.

## Spatial Sense and Geometric Concepts

- apply geometric properties, formulas, and relationships to solve real-world problems;
- justify conclusions using deductive reasoning;
- use inductive reasoning to make conjectures;
- communicate position using spatial sense with two- and three-dimensional coordinate systems.

# INTEGRATED MATHEMATICS I
## 1 Unit

**Course Number:** 3132

**Level:** Level 2 course

**Recommended Prerequisite:** Successful completion of 8th grade mathematics, Foundations I, and /or Foundations II.

**Emphasis:** Through the investigation of meaningful problems individually or in cooperative groups, while using appropriate technology, students will meet the learning expectations listed below.

## Learning Expectations:

### Number Sense and Number Theory

- demonstrate an understanding of the elements, subsets, properties, and operations of rational numbers;
- demonstrate understanding of positive integer exponents and perform operations with expressions involving exponents;
- connect physical, graphical, verbal, and symbolic representations of rational numbers;
- connect physical, graphical, verbal, and symbolic representations of absolute value;
- articulate, model, and apply the concept of inverse (i.e. opposites and reciprocals);
- describe, model, and apply inverse operations.

### Estimation, Measurement, and Computation

- choose appropriate techniques and tools to measure quantities in order to meet specifications for precision and accuracy;
- select and apply an appropriate method (i.e. mental arithmetic, paper and pencil, or technology) for computing with real numbers, and evaluate the reasonableness of results;
- communicate the concepts and strategies being used in estimation, measurement, and computation;
- perform operations on algebraic expressions and informally justify the procedures chosen;

- use concepts of length, area, and volume to estimate and solve real-world problems;
- apply measurement concepts and relationships in algebraic and geometric problem-solving situations;
- use estimation to make predictions and determine reasonableness of results;
- demonstrate an understanding of rates and other derived and indirect measurements (e.g. velocity, miles per hour, rpm, cost per unit);
- apply matrix addition, subtraction, and scalar multiplication in real-world problems (e.g. inventory), using appropriate technology.

## Patterns, Functions, and Algebraic Thinking

- solve linear equations and inequalities in one variable and represent the solution set symbolically, graphically, and verbally;
- communicate the meaning of variables in algebraic expressions, equations, and inequalities.
- identify dependent and independent variables in real-world situations;
- apply the concept of variable in simplifying algebraic expressions, solving equations, and solving inequalities
- interpret graphs that depict real-world phenomena;
- model real-world phenomena using graphs;
- represent functions with equations, graphs, tables, and words;
- understand and apply slope as rate of change;
- solve real-world problems represented by linear functions and interpret the slope and intercepts;
- solve systems of two equations in two unknowns using a variety of techniques;
- recognize and extend numerical, geometric, and spatial patterns;
- generalize numerical, geometric patterns verbally and symbolically.

## Statistics and Probability

- collect, represent, and describe linear and nonlinear data sets developed from the real world using appropriate technology;
- choose, construct, and analyze appropriate graphical representations for a data set;
- interpret data using the appropriate measure of central tendency for the data set;
- determine the measures of dispersion of a data set including range and quartiles;
- apply basic counting principles, introducing factorial notation;
- apply experimental and theoretical probability with simulations where appropriate;
- make predictions from a linear data set using a line of best fit.

## Spatial Sense and Geometric Concepts

- apply inductive reasoning in making conjectures, then test conjectures and/or determine a counterexample;
- apply properties of special pairs of angles (e.g. supplementary, complementary, vertical, and adjacent);
- articulate relationships of angles formed when parallel lines are cut by a transversal;
- apply the concept of slope to parallel and perpendicular lines;
- solve real world problems involving length, perimeter, and circumference;
- apply the properties of congruence and similarity to solve problems;
- use appropriate measurement techniques and tools in investigating properties of polygons (triangle angle properties, angles of polygons, and triangle inequalities).

# INTEGRATED MATHEMATICS II
## 1 Unit

**Course Number:** 3133

**Level:** Level 3 course

**Recommended Prerequisite:** Integrated Mathematics I

**Emphasis:** Through the investigation of meaningful problems individually or in cooperative groups, while using appropriate technology, students will meet the learning expectations listed below.

## Learning Expectations:

### Number Sense and Number Theory

- demonstrate an understanding of the elements, properties and operations of real numbers;
- demonstrate an understanding of the relative size of rational and irrational numbers;
- connect physical, graphical, verbal, and symbolic representations of real numbers;
- articulate, model and apply the concept of inverse (powers and roots);
- recognize the existence of imaginary numbers.

### Estimation, Measurement, and Computation

- choose appropriate techniques and tools to measure quantities in order to meet specifications for tolerance;
- select and apply an appropriate method (i.e. mental arithmetic, paper and pencil, or technology) for computing with real numbers, and evaluate the reasonableness of results;
- perform operations on algebraic expression and informally justify the procedures chosen;
- use concepts of length, area, and volume to estimate and solve real-world problems;
- apply measurement concepts and relationships in algebraic and geometric problem-solving situations;
- use estimation to make predictions and determine reasonableness of results;

- demonstrate an understanding of rates and other derived and indirect measurements (e.g. velocity, miles per hr, rpm, cost per unit);
- apply geometric properties in constructions using a variety of tools (e.g. paper folding, geometric software, reflections tools);
- apply matrix operations to solve real-world problems, using appropriate technology.

## Patterns, Functions, and Algebraic Thinking

- solve systems of three equations and three unknowns using a variety of techniques including inverse matrices with technology;
- describe the domain and range of a function;
- represent real-world problems involving sets, their intersections, union, and complements using Venn diagrams;
- apply Venn diagrams in problem solving;
- solve quadratic equations and inequalities using appropriate methods;
- solve radical equations using appropriate methods;
- graph absolute value functions and quadratic functions with emphasis on transformations;
- solve real-world problems modeled by absolute value or quadratic functions;
- recognize the conic sections from given information;
- recognize, extend, and create numerical, geometric, and spatial patterns;
- generalize patterns verbally and symbolically using function notation.

## Spatial Sense and Geometric Concepts

- demonstrate an understanding of geometric transformations (i.e. reflection, translation, rotation, and dilation);
- apply deductive reasoning using postulates and theorems to prove conclusions from given hypotheses;
- determine the truth of an implication, its converse, inverse, and contrapositive;
- apply right triangle properties, including geometric mean, The Pythagorean Theorem, special right triangles, and the trigonometric ratios;
- derive the distance formula for the distance between two points in a rectangular coordinate system;
- apply the distance and midpoint formulas in solving problems;
- solve real-world problems involving area with two- and three-dimensional shapes. Use coordinates to describe position in two and three dimensions.

## Statistics and Probability

- demonstrate an understanding of different sampling methods and when each is appropriate;
- use simulations to demonstrate probability experiments;
- use a variety of techniques to determine equations of best fit for quadratic data sets;
- analyze the validity of statistical conclusions;
- determine the probability of an event;
- determine the probability of mutually exclusive events.

# INTEGRATED MATHEMATICS III
## 1 Unit

**Course Number:** 3134

**Level:** Level 4 course

**Recommended Prerequisite:** Integrated Mathematics II

**Emphasis:** Through the investigation of meaningful problems individually or in cooperative groups, while using appropriate technology, students will meet the learning expectations listed below.

## Learning Expectations:

### Number Sense and Number Theory

- demonstrate an understanding of the laws of exponents, including integral and rational exponents;
- demonstrate an understanding of the elements, subsets, and properties of the complex number system.

### Estimation, Measurement, and Computation

- select and apply an appropriate method (i.e. mental arithmetic, paper and pencil, or technology) for computing with real numbers, and evaluate the reasonableness of results;
- perform operations on algebraic expression and justify the procedures chosen;
- use concepts of length, area, and volume to estimate and solve real-world problems;
- apply measurement concepts and relationships in algebraic and geometric problem-solving situations;
- use estimation to make predictions and determine reasonableness of results;
- demonstrate an understanding of rates and other derived and indirect measurements (e.g. velocity, miles per hr, rpm, cost per unit);
- perform operations on complex numbers of the form a + bi.

## Patterns, Functions, and Algebraic Thinking

- perform operations on functions, including composition, and determine the effects of the composition on the domain and range;
- demonstrate an understanding of the inverse of a function and determining if the inverse is a function;
- identify and describe the characteristics of families of functions;
- articulate the results of varying parameters of a parent function;
- solve polynomial equations and inequalities using appropriate technology;
- solve absolute value equations and inequalities;
- graph polynomial, exponential, and logarithmic and rational functions;
- solve exponential, logarithmic, and rational equations using appropriate methods and technology;
- solve real-world problems modeled by polynomial, exponential, logarithmic, and periodic functions;
- solve problems involving linear programming;
- demonstrate an understanding of recursive and explicit definitions of functions and sequences;
- recognize the difference between continuous and discrete situations;
- apply sigma notation with arithmetic and geometric series;
- represent a sequence using a list, graph, symbols, and words;
- determine an equation of a conic section from its graph.

## Spatial Sense and Geometric Concepts

- apply and justify properties of quadrilaterals and circles;
- solve real world problems involving volume of geometric solids;
- demonstrate an understanding of the Platonic Solids;
- demonstrate an understanding of uniqueness through indirect proofs;
- apply transformational matrices to transform geometric figures in a rectangular coordinate system.

## Statistics and Probability

- describe and apply the normal distribution and its properties;
- use z-scores to compare normally distributed data sets;
- use a variety of techniques to determine equations of best fit for nonlinear data sets;
- calculate and interpret z-scores;
- apply the properties of conditional probability;
- determine binomial probabilities using appropriate methods;
- make inferences about a data set using appropriate measures of central tendency and dispersion, including variance and standard deviation;
- calculate expected value to make judgments about real-life situations.

# ADVANCED ALGEBRA AND TRIGONOMETRY
## 1 Unit

**Course Number:** 3124

**Level:** Level 5 course

**Recommended Prerequisite:** Algebra I, Algebra II and Geometry or Integrated Mathematics I, II, and III**.**

**Emphasis:** Through the investigation of meaningful problems individually or in cooperative groups, while using appropriate technology, students will meet the learning expectations listed below.

## Learning Expectations:

### Applications of Trigonometry

- use degrees and radians interchangeably to represent angle measure in problems and explain the advantages/disadvantages of a particular choice;
- solve real-world problems applying the trigonometric ratios, the Law of Sines, and Law of Cosines;
- apply the trigonometric formulas for finding the area of triangles and circular sectors and segments;
- derive the Pythagorean Identities.

### Trigonometric Functions

- define the trigonometric functions using the unit circle;
- determine values of the trigonometric functions for special angles using the unit circle and the symmetry of the circle;
- graph the trigonometric functions;
- understand amplitude, period, phase shift, and vertical shift and apply to graphing trigonometric functions;
- model periodic phenomena using trigonometric functions with appropriate technology;
- verify trigonometric identities graphically and by substitution;
- solve trigonometric equations graphically.

## Understanding Functions

- represent and analyze a variety of functions and their characteristics graphically, algebraically, verbally, and numerically;
- graph a variety of functions using transformations;
- solve a variety of equations using appropriate methods;
- solve linear, quadratic, and polynomial inequalities using appropriate methods;
- solve real-world problems modeled by linear, quadratic, radical, rational, polynomial, exponential, and logarithmic functions;
- use data analysis techniques to model real-world phenomena using functions.

## Applications of Matrices

- demonstrate an understanding of operations on matrices;
- solve real-world problems involving networks, finite graphs, and geometric transformations.

## Sequences and Series

- recognize the difference between continuous and discrete situations;
- demonstrate an understanding of recursive and explicit definitions of functions and sequences;
- apply sigma notation with arithmetic and geometric series;
- represent a sequence using a list, graph, symbols, and words.

# DISCRETE MATHEMATICS WITH STATISTICS AND PROBABILITY
## 1 Unit

**Course Number:** 3135

**Level:** Level 5 course

**Recommended Prerequisite:** Algebra I, Algebra II and Geometry or Integrated Mathematics I, II, and III**.**

**Emphasis:** Through the investigation of meaningful problems individually or in cooperative groups, while using appropriate technology, students will meet the learning expectations listed below.

## Learning Expectations:

### Discrete Mathematics

- apply discrete ideas to solve real-world problems (i.e. election theory, group ranking, and estate planning);
- demonstrate an understanding of the elements, subsets, properties, operations of sets;
- use valid forms of deductive reasoning and logic to make and evaluate arguments;
- represent and solve problems using discrete structures such as finite graphs, matrices, and sequences (e.g. Leslie Model, Leontief Model, Markov Chain, and cryptographic techniques);
- use vertex-edge graphs to solve network problems such as finding circuits, critical paths, minimum spanning trees, and adjacency matrices;
- analyze and use discrete ideas such as induction, iteration, and recurrence relations to solve problems from such fields as Chaos Theory, Map Problems, and fractals;
- create tessellations using reflection, rotation, and translation.

### Probability

- apply number theory topics such as the Fundamental Theorem of Arithmetic, lowest common denominator, greatest common factor, etc. to solve problems using modular arithmetic;
- create and interpret discrete probability distributions;

- use experimental or theoretical probability, as appropriate, to represent and solve problems involving uncertainty;
- derive and use formulas to calculate combinations and permutations;
- understand and apply the concept of a random variable to generate and interpret probability distributions including binomial, uniform, normal, and Chi Square;
- apply game theory to problem solving.

**Statistics**

- design a statistical experiment to study a problem, conduct the experiment, and communicate and interpret the outcomes;
- understand and apply measures of central tendency, variability, and correlation to summarize data and draw inferences from real-world situations;
- understand sampling and recognize its role in statistical claims;
- conduct and interpret tests for significance using appropriate statistics;
- use curve fitting to make predictions from data;
- construct and draw inferences from charts, tables, and graphs that summarize data from real-world situations.

# PRECALCULUS
## 1 Unit

**Course Number:** 3126

**Level:** Level 5 course

**Recommended Prerequisite:** Algebra I, Algebra II and Geometry or Integrated Mathematics I, II, and III**.**

**Emphasis:** Through the investigation of meaningful problems individually or in cooperative groups, while using appropriate technology, students will meet the learning expectations listed below.

## Learning Expectations:

- model real-world phenomena using techniques of data analysis;
- recognize and apply mathematical models of linear, quadratic, exponential, logarithmic, and trigonometric functions;
- use scatterplot residuals, and/or correlation coefficients to determine whether a model is appropriate;
- use models when appropriate to draw conclusions or make predictions.

### Understanding Functions

- sketch the graphs of the basic functions (linear, quadratic, cubic, square root, absolute value, reciprocal, trigonometric, exponential, logarithmic, and greatest integer);
- graph transformations and combinations of transformations for all basic functions;
- analyze a function by decomposing it into simpler functions;
- determine if a function is even, odd, or neither;
- analyze functions;
- solve inequalities using an appropriate technology;
- introduce the concept of the limit of a function;
- apply the limit of a function to find the slope of a line tangent to a curve;
- apply limits to develop the concept of continuity;
- determine an equation of a rational function from a written description.

## Exponential and Logarithmic Functions

- define and use the logarithmic function as the inverse of the exponential function;
- sketch the graphs of exponential and logarithmic functions;
- solve exponential and logarithmic equations modeling real-world problems (e.g. growth and decay).

## Trigonometric Functions

- define six circular functions;
- sketch graphs of the six trigonometric functions involving period change, amplitude change, phase shift, and/or vertical shift;
- use trigonometric functions to model periodic phenomena;
- use graphs to develop and verify trigonometric identities;
- find values of inverse trigonometric functions, applying appropriate domain and range restrictions;
- solve trigonometric equations and inequalities either algebraically or using an automatic grapher.

## Applications of Trigonometry

- derive the Law of Sines and the Law of Cosines and apply them to solve problems involving triangles and vectors;
- derive and apply the formulas for the area of a triangle and the sector of a circle;
- understand the relationship between measurements in radians and degrees;
- apply radian measures in problems related to linear and angular velocity;
- understand and apply vectors to solve real world problems;
- represent complex numbers in both rectangular and polar form;
- apply the trigonometric form of complex number in calculations;
- prove and apply DeMoivre's Theorem to find roots and powers of complex numbers.

## Sequences and Series

- demonstrate an understanding of sequences by representing them recursively and explicitly;
- use sigma notation to represent a series;
- determine whether a given series converges or diverges;
- find the sum of an infinite series that converge;
- find the sum of an infinite geometric series.

## Conic Sections

- apply equations and graphs of conic sections to model real-world phenomena.

# STATISTICS
## 1 Unit

**Course Number:** 3136

**Level:** Level 5 course

**Recommended Prerequisite:** Algebra I, Algebra II and Geometry or Integrated Mathematics I, II, and III**.**

**Emphasis:** Through the investigation of meaningful problems individually or in cooperative groups, while using appropriate technology, students will meet the learning expectations listed below.

## Learning Expectations:

### Representing and Describing Data

- construct and interpret charts, tables, and graphs that display univariate and bivariate data;
- calculate and apply measures of central tendency and dispersion in order to make inferences about a data set;
- analyze the effects of data transformations on measures of central tendency and variability;
- calculate and apply the correlation between data sets.

### Analyzing Data

- apply the properties of a normal distribution or a Chi-square distribution in appropriate situations in order to make inferences about a data set;
- demonstrate an understanding of the Central Limit Theorem;
- use curve-fitting with appropriate technology to make regression equations in order to represent a data set algebraically and to make inferences;
- demonstrate an understanding of confidence intervals.

## Statistical Experiments

- demonstrate understanding of bias in sampling;
- demonstrate an understanding of the Law of Large Numbers;
- demonstrate an understanding of the probability of independent events and conditional probability;
- using appropriate probability models, design a method for simulating data from a particular situation, and use the generated data to analyze the situation;
- design and conduct a statistical experiment to study a problem, and interpret and communicate the outcomes;
- test hypotheses using appropriate statistics.

# CALCULUS
## 1 Unit

**Course Number:** 3113

**Level:** Level 5 course

**Recommended Prerequisite:** Advanced Algebra & Trigonometry and/or PreCalculus

**Emphasis:** Through the investigation of meaningful problems individually or in cooperative groups, while using appropriate technology, students will meet the learning expectations listed below.

**Learning Expectations:**

### Functions, Graphs, and Limits

- analyze the graphs of polynomial, rational, radical, and transcendental functions using appropriate technology;
- predict and explain the observed local and global behavior of a function;
- calculate limits using algebra;
- estimate limits from graphs or tables of data.

### Asymptotic and Unbounded Behavior

- demonstrate an understanding of asymptotes in terms of graphical behavior;
- describe asymptotic behavior in terms of infinite limits and limits at infinity;
- compare relative magnitudes of functions and their rates of change.

### Continuity as a Property of Functions

- demonstrate an understanding continuity in terms of limits;
- demonstrate a geometric understanding of graphs of continuous functions.

### Derivatives

- represent the concept of the derivative geometrically, numerically, and analytically;
- interpret the derivative as an instantaneous rate of change;

- define the derivative as the limit of the difference quotient;
- articulate the relationship between differentiability and continuity.
- articulate corresponding characteristics of graphs of $f$ and $f'$ ;
- communicate the relationship between the increasing and decreasing behavior $f$ and the sign of $f'$ ;
- demonstrate an understanding of the Mean Value Theorem and its geometric consequence;
- translate verbal descriptions into equations involving derivatives and vice versa.

## Second Derivatives

- articulate corresponding characteristics of the graphs of $f$ , $f'$ , and $f''$ ;
- communicate the relationship between the concavity of $f$ and the sign of $f''$ ;
- identify points of inflection.

## Applications of Derivatives

- analyze curves using the notions of monotonicity and concavity; optimization, both absolute (global) and relative (local) extrema;
- model rates of change, including related rates problems;
- use implicit differentiation to find the derivative of an inverse function;
- interpret the derivative as a rate of change in varied applied contexts.

## Computation of Derivatives

- apply basic rules for the derivative of basic functions and their sum, product, and quotient;
- use the chain rule and implicit differentiation.

## Integrals

- communicate the relationship between a Riemann summ and a definite integral;
- apply basic properties of definite integrals.

## Fundamental Theorem of Calculus

- evaluate definite integrals using the Fundamental Theorem.

## Antidifferentiation

- apply techniques of antidifferentiation;
- find specific antiderivatives using initial conditions, including applications to motion along a line;
- use separable differential equations in modeling.

## Numerical Approximations to Definite Integrals

- use Riemann sums and the Trapezoidal Rule to approximate definite integrals of functions represented algebraically, geometrically, and by tables of values.